PAP Algebra
Chapter 12 Test Review
Solving Exponential/Log Equations (Common Base or by Converting)

1) \(2^x = 16\)
   \[2^x = 2^4\]
   \[x = 4\]

2) \(4^{3x} = \left(\frac{1}{8}\right)^{2x+1}\)
   \[2^{(3x)} = 2^{-3(2x+1)}\]
   \[6x = -6x - 3\]
   \[12x = -3\]
   \[x = -\frac{1}{4}\]

3) \(3(e)^x + 3 = 9\)
   \[-3 - 3\]
   \[\frac{2e^x}{3} = \frac{6}{3}\]
   \[e^x = 2\]
   \[\ln 2 = x\] or \(\ln 2\) \(\approx 0.693\)

4) \(4(e)^{x+5} = 8\)
   \[\frac{4e^{x+5}}{4} = \frac{8}{4}\]
   \[e^{x+5} = 2\]
   \[\ln 5 = x + 5\]
   \[x = \ln(5) - 5\] or \(-3.39\)

5) \(\left(\frac{1}{3}\right)^x = 27\)
   \[3^{-x} = 3^3\]
   \[-x = 3\]
   \[x = -3\]

6) \(\log_2(x^2 + 3x - 10) = 3\)
   \[2^3 = x^2 + 3x - 10\]
   \[8 = x^2 + 3x - 10\]
   \[0 = x^2 + 3x - 18\]
   \[x = -6\]
   \[x = 3\]

7) \(3\ln(x + 5) = 21\)
   \[\frac{3}{3} = 3\]
   \[\ln(x + 5) = 7\]
   \[e^7 = x + 5\]
   \[x = e^7 - 5\] or \[x = 1091.633\]

8) \(2\log_3(4) - \log_3, x = \frac{2}{3}\log_3, 8\)
   \[\log_3 4 - \log_3 8\]
   \[\frac{1}{3} x = \log_3 4\]
   \[\frac{1}{3} x = \frac{4}{3}\]
   \[x = 4\]
9) \(14 + \log_7(x) = 16\)
\[
\begin{align*}
14 & \quad \text{14} \\
\log_7(x) & = 2 \\
7^2 & = x \\
x & = 49
\end{align*}
\]

10) \(\log_6(3x) - 10 = -8\)
\[
\begin{align*}
10 & \quad \text{10} \\
\log_6(3x) & = 2 \\
36 & = 3x \\
12 & = x \\
x & = 12
\end{align*}
\]

11) \(\ln(x) = 13 - \ln(x^2)\)
\[
\begin{align*}
\ln x + \ln x^2 & = 13 \\
\ln x^3 & = 13 \\
e^{13} & = x \\
3\sqrt{e^{13}} & = x \\
x & = 76.198
\end{align*}
\]

12) \(\log_3(x + 8) - \log_3(x - 4) = 2\)
\[
\begin{align*}
\log_3 \frac{x+8}{x-4} & = 2 \\
3 & = \frac{x+8}{x-4} \\
9x - 36 & = x + 8 \\
8x & = 44 \\
x & = 5.5
\end{align*}
\]

13) \(\log_4(4x) = 3 - \log_4(2x)\)
\[
\begin{align*}
\log_4(4x) + \log_4(2x) & = 3 \\
\log_4(8x^2) & = 3 \\
4^3 & = 8x^2 \\
\sqrt[8]{y} & = \frac{8}{y} \\
x & = 2\sqrt{2} \text{ or } 2.83
\end{align*}
\]

14) \(2\log_3(x) - \log_3(2) = 3\log_3(4)\)
\[
\begin{align*}
\log_3(x^2) - \log_3(2) & = \log_3(4)^3 \\
\log_3 \frac{x^2}{2} & = \log_3 64 \\
\frac{x^2}{2} & = 64 \\
\sqrt{\frac{x^2}{2}} & = \sqrt{128} \\
x & = 11.313
\end{align*}
\]

**Word Problems (Setting Up & Solving)**

15) Is it better to invest your money at 5.5% interest compounded continuously or at 5.8% interest compounded monthly if you have $12,000 to invest for 4 years?

\[
\begin{align*}
Y & = 12,000e^{0.055x} \\
Y & = 12,000e^{0.055(4)} \\
Y & = 14952.92
\end{align*}
\]

\[
\begin{align*}
Y & = 12,000(1 + \frac{0.058}{12})^{12x} \\
Y & = 12,000(1 + 0.0049)^{12(4)} \\
Y & = 15,124.98
\end{align*}
\]

Better option
16) $12,000 principal earning 4.8% interest after 4 years
   a.) Annually  b.) Semi-annually  c.) Quarterly  d.) Monthly
   \[ 12000 \left(1 + \frac{0.048}{1}\right)^4 = 14478.26 \]
   \[ 12000 \left(1 + \frac{0.048}{2}\right)^8 = 14507.109 \]
   \[ 12000 \left(1 + \frac{0.048}{4}\right)^{16} = 14523.438 \]
   \[ 12000 \left(1 + \frac{0.048}{12}\right)^{48} = 14534.978 \]

17) If you have an account that has an interest rate of 1.9% compounded monthly, how long will it take for your money to triple?
   \[ \frac{300}{100} = \frac{100}{100} \left(1 + \frac{0.019}{12}\right)^{12x} \]
   \[ 3 = \left(1 + \frac{0.019}{12}\right)^{12x} \rightarrow \frac{\log(1.019/12)}{\log(1.019/12)} = \frac{x}{12} \]
   \[ x = \frac{\log(3)}{\log(1.019/12)} \]
   \[ x = 57.4 \text{ months} \]

18) Mosquitoes are tripling in number each week. If there are currently 300 mosquitoes in your bug zapper in the back yard, when will there be 2000 mosquitoes?
   \[ \frac{2000}{300} = \frac{300}{300} \left(3\right)^x \]
   \[ 6^{2/3} = 3^x \]
   \[ \log_3(6^{2/3}) = x \text{ or } \log_3\left(\frac{2000}{300}\right) = x \]
   \[ x = \frac{\log(6^{2/3})}{\log(3)} \]
   \[ x = 1.731 \text{ weeks} \]

19) As a town gets smaller, the population of its high school decreases by 12% each year. The student body has 538 students now. In how many years will it have 390 students?
   \[ y = 538 \left(1 - 0.12\right)^x \]
   \[ \frac{390}{538} = 538 \left(0.88\right)^x \]
   \[ \log_538\left(\frac{390}{538}\right) = x \]
   \[ x = 2.52 \text{ years} \]

20) The world population in 2000 was approx. 6.08 billion. The annual rate of increase was about 1.26%. If the world population continues to grow at this rate, when will the population reach 9 billion?
   \[ q = \frac{9}{6.08} \]
   \[ q = 1.48 \]
   \[ \ln\left(\frac{9}{6.08}\right) = \ln\left(\frac{q}{q}\right) \]
   \[ x = 31 \text{ years} \]
21) Nobelium-259 has a half-life of 58 minutes. How much remains of a 1 kg sample after 1 day?

\[ y = 1 \left( \frac{1}{2} \right)^{\frac{1440}{58}} \]

\[ y = 3.359 \times 10^{-8} \]

22) Dubnium-262 has a half-life of 34 seconds. How many grams did we begin with if, after 5 minutes, we are left with only 1 gram?

\[ 1 = x \left( \frac{1}{2} \right)^{\frac{300}{27}} \]

\[ x = 453.1 \]

23) Your parents bought a boat for $60,000. The boat will depreciate by 9.3% each year. When will the boat be worth $10,000?

\[ \frac{10,000}{60,000} = \left( 1 - 0.093 \right)^x \]

\[ \frac{1}{6} = (1 - 0.093)^x \]

\[ \log_{0.907} \frac{1}{6} = x \]

\[ x = 18.36 \]

24) If a population of 175 red-spotted toads doubles every 2 years, how many toads can you expect to find in 10 years?

\[ y = 175 \left( \frac{1}{2} \right)^{\frac{10}{2}} \]

\[ y = 3600 \]

25) A student wants to have $8000 for college 5 years from now. How much should she put into an account that earns 5.2% annual interest compounded continuously?

\[ \frac{8,000}{e^{0.052(5)}} = x \]

\[ 6,158.41 = x \]