

Solving Exponential and Logarithm Equations, Day 1

Under each number, rewrite using exponentials:

4	8	9	16	25	27	32	36	64	81	125	1/4	1/8
2^2	2^3	3^2	4^2	5^2	3^3	2^5	6^2	8^2 2^6	9^2 3^4	5^3	2^{-2}	2^{-3}

Solving by Common Base: Exponentials

If possible, rewrite the equation where both sides are expressed as powers of the same base. Then, the exponents will be equal to each other and you can solve.

1. $7^{6x} = 7^{2x-20}$

$$\begin{array}{r} 6x = 2x - 20 \\ -2x \quad -2x \\ \hline 4x = -20 \end{array}$$

$$4x = -20$$

$$x = -5$$

3. $2^{2x} = 8^{2x-1}$

$$2^{2x} = 2^{3(2x-1)}$$

$$\begin{array}{r} 2x = 6x - 3 \\ -6x \quad -6x \\ \hline -4x = -3 \end{array}$$

$$x = \frac{3}{4}$$

5. $9^{x-4} = 1/81$

$$9^{x-4} = 9^{-2}$$

$$\begin{array}{r} x-4 = -2 \\ +4 \quad +4 \\ \hline x = 2 \end{array}$$

$$x = 2$$

Solving by Common Base: Logarithms

If possible, write both sides of the equation with a common base and set the arguments (power) equal to each other.

7. $\log_5(x+7) = \log_5(2x-3)$

$$\begin{array}{r} x+7 = 2x-3 \\ -x \quad -x \\ \hline 7 = x-3 \end{array}$$

$$\begin{array}{r} 7 = x-3 \\ +3 \quad +3 \\ \hline 10 = x \end{array}$$

$$x = 10$$

2. $e^{2x-7} = e^{5x+4}$

$$\begin{array}{r} 2x-7 = 5x+4 \\ -2x \quad -2x \\ \hline -7 = 3x+4 \end{array}$$

$$\begin{array}{r} -7 = 3x+4 \\ -4 \quad -4 \\ \hline -11 = 3x \end{array}$$

$$x = -11/3$$

4. $4^x = 32$

$$2^{2x} = 2^5$$

$$2x = 5$$

$$x = 5/2$$

6. $25^{10x+8} = (1/125)^{4-2x}$

$$5^{2(10x+8)} = 5^{-3(4-2x)}$$

$$\begin{array}{r} 20x+16 = -12+6x \\ -6x \quad -6x \\ \hline 14x+16 = -12 \end{array}$$

$$\begin{array}{r} 14x+16 = -12 \\ -16 \quad -16 \\ \hline 14x = -28 \\ 14x = -28 \\ \hline x = -2 \end{array}$$

8. $\log_4(3x+2) = \log_4(6-x)$

$$\begin{array}{r} 3x+2 = 6-x \\ +x \quad +x \\ \hline 4x+2 = 6 \end{array}$$

$$\begin{array}{r} 4x+2 = 6 \\ -2 \quad -2 \\ \hline 4x = 4 \end{array}$$

$$4x = 4$$

$$x = 1$$

If the common base is not possible, use log properties and write your equation as a single log and convert to an exponential. If exponential then isolate and convert to a logarithm and solve.

9. $\log_4(3x-5) = 3$

$$4^3 = 3x-5$$

$$64 = 3x-5$$

$$\begin{array}{r} +5 \\ \hline 69 = 3x \end{array}$$

$$x = 23$$

10. $\ln(x-1) - \ln x = 2$

$$\ln \frac{x-1}{x} = 2$$

$$e^2 = \frac{x-1}{x}$$

$$\frac{7.389}{1} = \frac{x-1}{x}$$

$$\begin{array}{r} 7.389x = x-1 \\ -x \quad -x \\ \hline 6.389x = -1 \end{array}$$

$$\frac{6.389x}{6.389} = \frac{-1}{6.389}$$

$$x = \frac{-1}{6.389}$$

11. $\log_4(x-3) + \log_4(x+3) - \log_4(x+3) = 3$

$$\log_4 \left(\frac{(x-3)(x+3)}{(x+3)} \right) = 3$$

$$\log_4(x-3) = 3$$

$$4^3 = x-3$$

$$64 = x-3$$

$$\begin{array}{r} +3 \quad +3 \\ \hline x = 67 \end{array}$$

12. $5 - 3e^x = 2$

$$\begin{array}{r} -5 \quad -5 \\ \hline -3e^x = -3 \end{array}$$

$$\frac{-3e^x}{-3} = \frac{-3}{-3}$$

$$e^x = 1$$

$$\ln 1 = x$$

$$x = 0$$

13. $e^{\ln x} = 4$

$$\ln x \cdot e = 4$$

$$\ln_e e^x = 4$$

$$x = 4$$

14. $(6)^{-4x} + 1 = 6$

$$\begin{array}{r} -1 \quad -1 \\ \hline 6^{-4x} = 5 \end{array}$$

$$\frac{\log_6(5)}{-4} = \frac{-4x}{-4}$$

$$x = \frac{\log_6(5)}{-4}$$

Alpha Window #5

$$x = -.225$$

15. $6 + 3\ln x = 4$

$$\begin{array}{r} -6 \quad -6 \\ \hline 3\ln x = -2 \end{array}$$

$$\frac{3\ln x}{3} = \frac{-2}{3}$$

$$\ln_e x = -2/3$$

$$e^{-2/3} = x$$

Solving Exponential and Log Equations (Day 1)

Solve for x. Round answers to the nearest thousandth.

1) $\log_7(2-x) = \log_7 5x$

$$2-x = 5x$$

$$6x = 2$$

$$\boxed{x = 1/3}$$

2) $\log_6(x+3) - \log_6(x-1) = \log_6 7$

$$\log_6 \frac{x+3}{7(x-1)} = 0$$

$$6^0 = \frac{x+3}{7(x-1)}$$

$$1 = \frac{x+3}{7(x-1)}$$

$$x+3 = 7(x-1)$$

$$6x = 10$$

$$\boxed{x = 5/3}$$

$$\log_6 \frac{x+3}{x-1} = \log_6 7$$

$$\frac{x+3}{x-1} = 7$$

$$x+3 = 7(x-1)$$

$$x+3 = 7x-7$$

$$10 = 6x$$

$$\boxed{x = \frac{10}{6} = \frac{5}{3}}$$

3) $8 + 4 \ln x = 6$

$$4 \ln x = -2$$

$$\ln x = -1/2$$

$$\boxed{e^{-1/2} = x}$$

4) $2 \log_3 4x = 6$

$$\log_3(4x) = 3$$

$$3^3 = 4x$$

$$\frac{27}{4} = \frac{4x}{4}$$

$$\boxed{x = \frac{27}{4}}$$

5) $9^{x-4} = \left(\frac{1}{81}\right)$

$$9^{x-4} = 9^{-2}$$

$$\begin{array}{r} x-4 = -2 \\ +4 \quad +4 \end{array}$$

$$\boxed{x = 2}$$

6) $16 - 7e^x = 2$

$$-7e^x = -14$$

$$e^x = 2$$

$$\boxed{\ln 2 = x}$$

$$7) 5(3^{x-1}) = 25$$

$$3^{x-1} = 5$$

$$\log_3 5 = x - 1$$

$$x = \log_3(5) + 1$$

$$8) e^{2x-7} = e^{5x+4}$$

$$2x - 7 = 5x + 4$$

$$-\frac{11}{3} = \frac{3x}{3}$$

$$x = -\frac{11}{3}$$

$$9) 5^{\sqrt{x}} = 25$$

$$5^{\sqrt{x}} = 5^2$$

$$(\sqrt{x}) = 2$$

$$x = 4$$

$$10) 8^{x+1} = 32$$

$$2^{3(x+1)} = 2^5$$

$$3x + 3 = 5$$

$$-3 \quad -3$$

$$3x = 2$$

$$x = \frac{2}{3}$$

$$11) \ln(x+2) - \ln(4) = 3$$

$$\ln \frac{x+2}{4} = 3$$

$$e^3 = \frac{x+2}{4}$$

$$4e^3 = x+2$$

$$x = 4e^3 - 2$$

$$12) \log_3(5x+13) - \log_3(3x) = \log_3 6$$

$$\log_3 \frac{5x+13}{3x} = \log_3 6$$

$$\frac{5x+13}{3x} = \frac{6}{1}$$

$$5x+13 = 18x$$

$$-5x \quad -5x$$

$$13 = 13x$$

$$x = 1$$

$$\log_3 \frac{5x+13}{6(3x)} = 0$$

$$3^0 = \frac{5x+13}{6(3x)}$$

$$1 = \frac{5x+13}{18x}$$

$$18x = 5x+13$$

$$13x = 13$$

$$x = 1$$