Name: Key Period: ______ Date: ____________________

Notes: Solving Absolute Value Equations and Inequalities

efine Absolute Value: Distance Away from zero

Solving Absolute Value Equations:

Since the value of x inside the absolute value bars could be a positive or negative number, account for both possible solutions. To do so, set the right side of = to be a positive and negative number to solve for both solutions.

Example 1: Solve $|x + 5| = 7$

$$\begin{array}{l} x+5 = 7 \\ -5 \quad -5 \\ \hline x = 2 \end{array} \qquad \begin{array}{l} x+5 = -7 \\ -5 \quad -5 \\ \hline x = -12 \end{array}$$

Check: $|2+5| = 7$, $7 = 7$ ✓ $\qquad |-12+5| = 7$, $|-7| = 7$ ✓

Extraneous Solutions

You must check your solutions to see if they may be extraneous (solutions that do not work when plugged back into equation.) Check your answers from Example 1 to see if you have any extraneous solutions.

Example 2: Solve $|5x + 7| = 3x - 4$

$$\begin{array}{l} 5x+7 = 3x-4 \\ -3x \qquad -3x \\ \hline 2x+7 = -4 \\ \quad -7 \quad -7 \\ \hline \frac{2x}{2} = \frac{-11}{2} \\ \boxed{x = -5.5} \end{array} \qquad \begin{array}{l} 5x+7 = -(3x-4) \\ 5x+7 = -3x+4 \\ +3x \qquad +3x \\ \hline 8x+7 = 4 \\ \quad -7 \quad -7 \\ \hline \frac{8x}{8} = -\frac{3}{8} \\ \boxed{x = -3/8} \end{array}$$

Isolate Abs 1st

Example 3: Solve $3|x+6|+6=9x$

$3|x+6|+6=9x$
$-6 \quad -6$

$\frac{3|x+6|}{3}=\frac{9x-6}{3}$

$x+6=3x-2$
$-x+2 \quad -x+2$
$\frac{8}{2}=\frac{2x}{2}$
$\boxed{x=4}$ ✓

$x+6=-3x+2$
$-x-2 \quad -x-2$
$\frac{4}{-4}=\frac{-4x}{-4}$
$\boxed{x=-1}$ → extraneous

$3|-1+6|+6=9(-1)$
$3|5|+6=-9$
$15+6\neq-9$

Practice

Solve the following absolute value equations. Check for extraneous solutions.

1. $|2x-5|=13$

$2x-5=13$
$+5 \quad +5$
$2x=18$
$\boxed{x=9}$

$2x-5=-13$
$+5 \quad +5$
$2x=-8$
$\boxed{x=-4}$

2. $|7x-10|=4$

$7x-10=4$
$+10 \quad +10$
$7x=14$
$\boxed{x=2}$

$7x-10=-4$
$+10 \quad +10$
$\frac{7x}{7}=\frac{6}{7}$
$\boxed{x=6/7}$

3. $|20-9x|=7$

$20-9x=7$
$-20 \quad -20$
$\frac{-9x}{-9}=\frac{-13}{-9}$
$\boxed{x=\frac{13}{9}}$

$20-9x=-7$
$-20 \quad -20$
$-9x=-27$
$\boxed{x=3}$

4. $5|x+4|-2=43$
$+2 \quad +2$

$\frac{5|x+4|}{5}=\frac{45}{5}$

$x+4=9$
$-4 \quad -4$
$\boxed{x=5}$

$x+4=-9$
$-4 \quad -4$
$\boxed{x=-13}$

5. $\frac{2|2x+4|}{2}=\frac{44}{2}$

$2x+4=22$
$-4 \quad -4$
$2x=18$
$\boxed{x=9}$

$2x+4=-22$
$-4 \quad -4$
$\frac{2x}{2}=\frac{-26}{2}$
$\boxed{x=-13}$

6. $|3x-4|=x+1$

$3x-4=x+1$
$-x+4 \quad -x+4$
$2x=5$
$\boxed{x=5/2}$

$3x-4=-x-1$
$+x+4 \quad +x+4$
$\frac{4x}{4}=\frac{3}{4}$
$\boxed{x=3/4}$

Solving Absolute Value Inequalities

Like absolute value equations, absolute value inequalities could also have two solutions. When solve inequalities, the first inequality is solved normally, and with the second inequality, you must make the right side of the symbol negative and flip the direction of the inequality symbol.

Example 4: Solve $|4x + 5| > 13$

$$4x+5>13$$
$$-5 \quad -5$$
$$4x>8$$
$$\boxed{x>2}$$

$$4x+5<-13$$
$$-5 \quad -5$$
$$\frac{4x}{4}<\frac{-18}{4}$$
$$\boxed{x<-9/2}$$

Example 5: Solve $|2x - 7| \le 21$

$$2x-7\le 21$$
$$+7 \quad +7$$
$$\frac{2x}{2}\le\frac{28}{2}$$
$$\boxed{x\le 14}$$

$$2x-7\ge -21$$
$$+7 \quad +7$$
$$\frac{2x}{2}\ge\frac{-14}{2}$$
$$\boxed{x\ge -7}$$

Practice

Solve the following absolute value inequalities.

7. $|x + 2| \ge 13$

$$x+2\ge 13$$
$$-2 \quad -2$$
$$\boxed{x\ge 11}$$

$$x+2\le -13$$
$$-2 \quad -2$$
$$\boxed{x\le -15}$$

8. $|2x + 1| > 9$

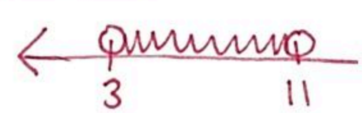
$$2x+1>9$$
$$-1 \quad -1$$
$$2x>8$$
$$\boxed{x>4}$$

$$2x+1<-9$$
$$-1 \quad -1$$
$$2x<-10$$
$$\boxed{x<-5}$$

9. $|7 - x| < 4$

$$7-x<4$$
$$-7 \quad -7$$
$$-x<-3$$
$$\boxed{x>3}$$

$$7-x>-4$$
$$-7 \quad -7$$
$$-x>-$$
$$\boxed{x<11}$$



10. **The Alberta Clipper**

When a fast moving "Alberta Clipper" cold-front drops down from Alberta, Canada and sweeps across a relatively small path through the Northeast quarter of the US, the temperature drops quickly, often by as much as 25° (and sometimes more!), and then recovers quickly. The given data representatioı lists the temperature in Columbus, Ohio from midnight (t = 0) to 10 in the morning during an "Alberta Clipper." The front arrived on the previous day. On that day, the daily high was 18° at 4p.m.

vertex

t (hours)	0	1	2	3	4	5	6	7	8	9	10
T	2	0	-2	-4	-6	-8	-6	-4	-2	0	2

"Chem y for Slope"

-2 -2 -2

A. If a meteorologist wants to model this data with a function, what function can be used?

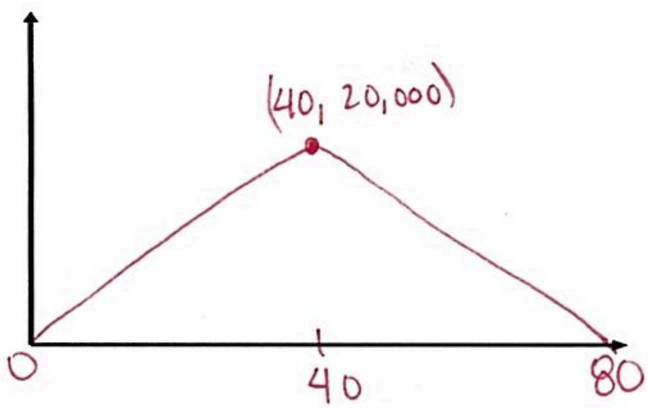
$y = -2|x-5|-8$

11. **Airplane Flight Model**

Slope

An airplane takes off from DFW airport and climbs at a constant rate of 500 ft/minute for forty minutes. It then descends at a constant rate of 500 ft/minute for the next forty minutes to land in Houston.

A. Graph the situation in the airplane model.



B. Write the equation for the airplane model.

$y = -500|x-40|+20{,}000$

Slope: 500

Right 40

Up 20,000

500(40)

Name: Key Period: ______ Date: ____________________

Practice: Solving Absolute Value Equations and Inequalities

Solve the following absolute value equations and inequalities. Be sure to check for extraneous solutions, if applicable.

1. $|4x - 7| < 9$

$4x - 7 < 9$
$+7 \quad +7$
$4x < 16$
$\boxed{x < 4}$

$4x - 7 > -9$
$+7 \quad +7$
$4x > -2$
$\boxed{x > -\tfrac{1}{2}}$

$-\tfrac{1}{2}$ $\quad$ 4

2. $|1 - 2x| = 9$

$1 - 2x = 9$
$-1 \quad -1$
$-2x = 8$
$\boxed{x = -4}$

$1 - 2x = -9$
$-1 \quad -1$
$-2x = -10$
$\boxed{x = 5}$

3. $|5 - 6x| + 3 = 10$
$-3 \quad -3$

$5 - 6x = 7$
$-5 \quad -5$
$-6x = 2$
$x = -\frac{2}{6}$
$\boxed{x = -\tfrac{1}{3}}$

$5 - 6x = -7$
$-5 \quad -5$
$-6x = -12$
$\boxed{x = 2}$

4. $\left|\frac{1}{3}x + 4\right| > 1$

$\frac{1}{3}x + 4 > 1$
$-4 \quad -4$
$3 \cdot \frac{1}{3}x > -3 \cdot 3$
$\boxed{x > -9}$

$\frac{1}{3}x + 4 < -1$
$-4 \quad -4$
$3 \cdot \frac{1}{3}x < -5 \cdot 3$
$\boxed{x < -15}$

-15 $\quad$ -9

5. $\frac{2|2x + 6|}{2} = \frac{24}{2}$

$2x + 6 = 12$
$-6 \quad -6$
$2x = 6$
$\boxed{x = 3}$

$2x + 6 = -12$
$-6 \quad -6$
$2x = -18$
$\boxed{x = -9}$

6. $|2 - 3x| \geq \frac{2}{3}$

$2 - 3x \geq \frac{2}{3}$
$-2 \quad -2$
$\frac{-3x}{-3} \geq \frac{-\frac{4}{3}}{-3}$
$\boxed{x \leq \frac{4}{9}}$

$2 - 3x \leq -\frac{2}{3}$
$-2 \quad -2$
$\frac{-3x}{-3} \leq \frac{-\frac{8}{3}}{-3}$
$\boxed{x \geq \frac{8}{9}}$

$4/9$ $\quad$ $\frac{8}{9}$

7. $\frac{1}{2}\left|\frac{2}{3}x + 2\right| = 0$

$\frac{2}{3}x + 2 = 0$
$-2 \quad -2$
$\cancel{\frac{3}{2}} \cdot \frac{2}{3}x = -2 \cdot \frac{3}{2}$
$x = -\frac{6}{2}$
$\boxed{x = -3}$

8. $2|3x - 3| - 2 = 14$
$+2 \quad +2$
$\frac{\cancel{2}|3x - 3|}{\cancel{2}} = \frac{16}{2}$

$3x - 3 = 8$
$+3 \quad +3$
$\frac{3x}{3} = \frac{11}{3}$
$\boxed{x = 11/3}$

$3x - 3 = -8$
$+3 \quad +3$
$\frac{3x}{3} = \frac{-5}{3}$
$\boxed{x = -5/3}$

9. $\left|4 - \frac{1}{2}x\right| \leq 6$

$4 - \frac{1}{2}x \leq 6$
$-4 \quad -4$
$(-2) - \frac{1}{2}x \leq 2(-2)$
$\boxed{x \geq -4}$

$4 - \frac{1}{2}x \geq -6$
$-4 \quad -4$
$(-2) - \frac{1}{2}x \geq -10$
$\boxed{x \leq +20}$

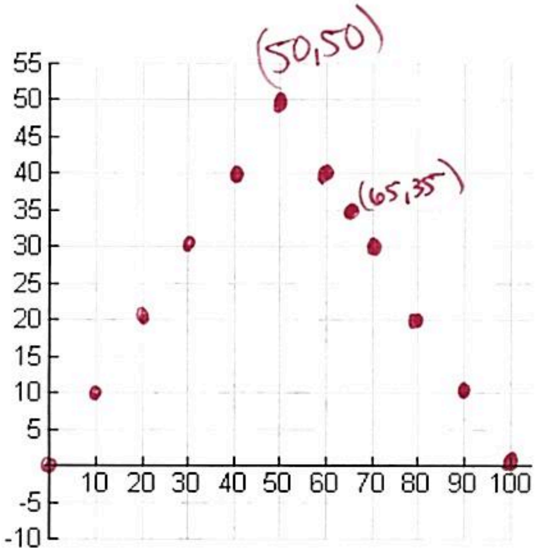
-4 $\quad$ 20

10. Yard lines of a football field have the relationship shown in the table below (0 yard lines are the goal lines).

FOOTBALL FIELD YARD LINES

Distance from one endzone (yd)	0	10	20	30	40	50	60	70	80	90	100
Marked yard line	0	10	20	30	40	50	40	30	20	10	0

Slope: $\frac{10-0}{10-0} = \frac{10}{10} = 1$



a. Write an absolute value function to find the marked yard line for a given distance from the endzone. (Hint: Graph the given ordered pairs to find the transformation from the parent function)

$y = -|x-50| + 50$

b. What yard line is 65 yards from the endzone?

$y = -|65-50| + 50$

$y = -(15) + 50$

$\boxed{y = 35}$