

Rational Functions

Simplifying Rational Expressions

$$1) \frac{x-2}{x^2+7x-18} \div \frac{x^3-6x^2-27x}{x^2+12x+27}$$

~~$x(x^2-6x-27)$~~
 ~~$(x-9)(x+3)$~~

$$\frac{x-2}{(x+9)(x-2)} \cdot \frac{(x+9)(x+3)}{x(x-9)(x+3)} = \frac{x-2}{x(x-9)(x-9)}$$

$$= \frac{1}{x(x-9)}$$

3x+1
3x
3x
12x+4

$$2) \frac{3x^2+13x+4}{x^2-4} \div \frac{4x+16}{x+2}$$

$$\frac{(3x+1)(x+4)}{(x+2)(x-2)} \cdot \frac{x+2}{4(x+4)} = \frac{3x+1}{4(x-2)}$$

$$3) \frac{x+5}{4x-16} \cdot \frac{2x^2-32}{x^2-25}$$

$\frac{(x+4)(x-4)}{2(x^2-16)}$

$$= \frac{2(x+4)}{4(x-5)}$$

$$= \frac{x+4}{2(x-5)}$$

$$4) \frac{(x-3)}{(x-3)(x-2)(x+1)} \cdot \frac{1}{x^2-x-2} \cdot \frac{x}{(x+1)}$$

~~x^2-5x+6~~
 ~~$(x-3)(x-2)(x+1)$~~
At. $x \neq 3, 2$

$$\frac{x-3 - [x(x+1)]}{(x-3)(x-2)(x+1)}$$

$$\frac{x-3 - x^2 - x}{(x-3)(x-2)(x+1)}$$

$$\frac{-x^2-3}{(x-3)(x-2)(x+1)}$$

$$5) \frac{x+4}{x^2-4} - \frac{15}{x-2(x+2)}$$

~~$15(x+2)$~~

$$\frac{x+4 - 15x - 30}{(x+2)(x-2)}$$

$$\frac{-14x - 26}{(x+2)(x-2)}$$

At. $x \neq -2, 2$

Solving Rational Equations

$$6) \frac{x}{x-1} + \frac{4x-3}{x-1}$$

$$x + x^2 - x = 4x - 3$$

$$x^2 - 4x + 3 = 0$$

$$(x-3)(x-1) = 0$$

$$\boxed{x=3}$$

~~$x=1 \rightarrow$ extraneous~~

$$7) 1 = \frac{1}{x^2+2x} + \frac{x-1}{x(x+2)}$$

$$\frac{1}{1} = \frac{1+x^2+x-2}{x(x+2)}$$

$$x^2+x-1 = x(x+2)$$

$$x^2 + \frac{-1}{-1} = x^2 + 2x - x$$

$$\boxed{-1 = x}$$

$$8) \frac{3x}{x^2-x-20} + \frac{2(x-5)}{x+4} = \frac{5x(x+4)}{x-5(x+4)}$$

$$3x + 2x - 10 = 5x^2 + 20x$$

$$0 = 5x^2 + 15x + 10$$

$$5(x^2 + 3x + 2)$$

$$5(x+2)(x+1)$$

$$\boxed{x = -2}$$

$$\boxed{x = -1}$$

Solving Rational Word Problems

- 9) Bob can paint a fence in 12 hours. Working with Sam they can paint a fence in 3 hours. How long does it take for Sam to work alone?

$$\left(\frac{x}{x}\right)\frac{1}{12} + \frac{1}{x}\left(\frac{12}{12}\right) = \frac{1}{3}\left(\frac{4x}{4x}\right)$$

$$\begin{array}{r} x + 12 = 4x \\ -x \quad \quad -x \\ \hline 12 = 3x \\ \boxed{x = 4} \end{array}$$

- 10) A boat can travel 6 miles an hour in still water. If it can travel 20 miles down a stream in the same time that it can travel 10 miles up the stream, what is the rate of the stream?

$$\frac{20}{6+x} = \frac{10}{6-x}$$

$$20(6-x) = 10(6+x)$$

$$\begin{array}{r} 120 - 20x = 60 + 10x \\ -60 + 20x \quad -60 + 20x \\ \hline \end{array}$$

$$\begin{array}{r} 60 = 30x \\ \frac{60}{30} = \frac{30x}{30} \\ \boxed{x = 2} \end{array}$$

- 11) Suppose one painter can paint the entire house in eighteen hours, and the second painter takes six hours. How long would it take the two painters together to paint the house?

$$\left(\frac{x}{x}\right)\frac{1}{18} + \frac{1}{6}\left(\frac{3x}{3x}\right) = \frac{1}{x}\left(\frac{18}{18}\right)$$

$$\begin{array}{r} x + 3x = 18 \\ 4x = 18 \\ \frac{4x}{4} = \frac{18}{4} \end{array}$$

$$\boxed{x = 4.5}$$

- 12) The manufacturer of cell phones has a fixed cost of \$35,000, plus a cost of \$27 per phone.

Average cost **per** phone equation: $y = \frac{27x + 35,000}{x}$

Range: $(27, 35,027)$

Find the average cost per phone if the manufacturer produces 1000 phones

$$y = \frac{27(1000) + 35,000}{1000}$$

$$\boxed{y = 62}$$

Radical Functions

Graphs & Attributes

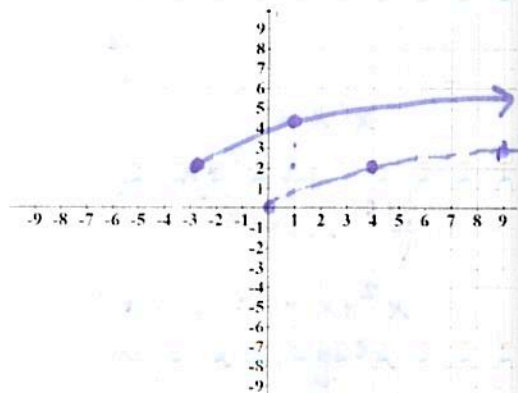
- 13) Given the function, $y = \sqrt{x+3} + 2$, answer the following:

Domain: $[-3, \infty)$

Range: $[2, \infty)$

maximum value: None

minimum value: $y = 2$



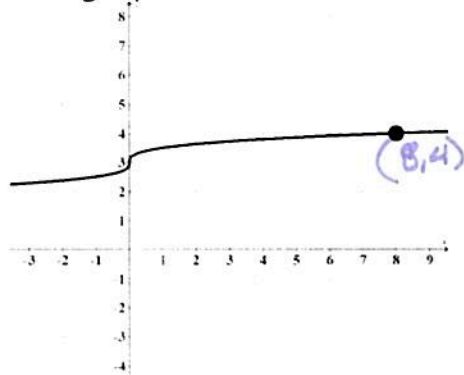
14) Given $y = \sqrt{x}$, write the equation of the graph that has been vertically compressed, shifted up 2 units, and reflected over the x-axis. Find the minimum or maximum value.

Equation: $y = -\frac{1}{2}\sqrt{x} + 2$ Minimum or Maximum: $y = 2$
(Circle One)

15) Given $y = \sqrt[3]{x}$, write the equation of the graph that has been horizontally compressed, shifted right 3 units, and down 7 units. Find the minimum or maximum value.

Equation: $y = \sqrt[3]{2(x-3)} - 7$ Minimum or Maximum: None
(Circle One)

16) Given the graphs find the function equation (make sure to find the a-value)



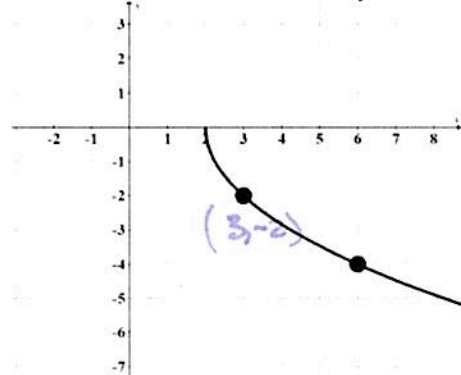
a.

$$y = a\sqrt[3]{x} + 3$$

$$4 = a\sqrt[3]{8} + 3$$

$$\frac{-3}{-3} = \frac{2a}{-3}$$

$$y = \frac{1}{2}\sqrt[3]{x} + 3$$



b.

$$y = a\sqrt{x-2}$$

$$-2 = a\sqrt{3-2}$$

$$-2 = a$$

$$y = -2\sqrt{x-2}$$

Simplifying: Simplify the following rational exponent expressions.

17) $(81xy)^{\frac{1}{4}}(16x^2y^2)^{\frac{3}{2}}$

$$3x^{1/4}y^{1/4} \cdot 64x^3y^3$$

$$192x^{13/4}y^{13/4}$$

18) $(8x^3)^{\frac{1}{3}}(2xy)^0$

$$16x^4$$

Solving: Solve the following equations.

19) $\sqrt[3]{2x-7} = 5$

$$\sqrt[3]{2x-7}^3 = 5^3$$

$$2x-7 = 125$$

$$\frac{2x}{2} = \frac{132}{2}$$

$$x = 66$$

21) $\sqrt{3x+1} - 1 = 6$

$$\sqrt{3x+1} = 7$$

$$3x+1 = 49$$

$$\frac{3x}{3} = \frac{48}{3}$$

$$x = 16$$

20) $(5x-4)^{\frac{3}{4}} = 27$

$$\sqrt[4]{(5x-4)^3} = 27$$

$$\sqrt[4]{5x-4} = 3$$

$$5x-4 = 81$$

$$\frac{5x}{5} = \frac{85}{5}$$

$$x = 17$$

22) $\sqrt{4x-48} = \sqrt{15x-103}$

$$4x-48 = 15x-103$$

$$-4x+103 = -4x+103$$

$$55 = 11x$$

$$\frac{55}{11} = \frac{11x}{11}$$

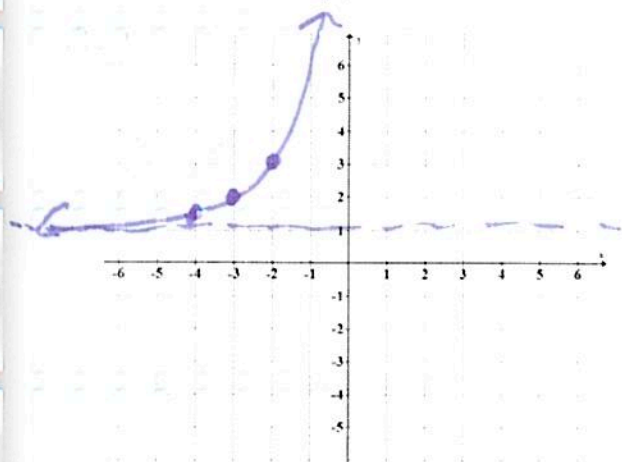
$$x = 5$$

Exponential Functions

Graphs and Key Attributes

23) Graph and identify the key attributes. (Use the tables if they help you. Asymptote and critical point must be shown on the graph.)

$$y = (2)^{x+3} + 1$$



x	y
-1	4/2
0	1
1	2

Table for $y = (2)^x$

-3	+1
-4	1.5
-3	2
-2	3

Apply Transformations for $y = (2)^{x+3} + 1$

Transformations: left 3; up!

Asymptote: $y = 1$

Domain: \mathbb{R}

Range: $y > 1$

24) Identify the key attributes given the following function:

$$y = -4\left(\frac{1}{2}\right)^x - 6$$

Transformations: V. Reflection
V. Stretch
Down 6

Critical Point: $(0, -10)$

Asymptote: $y = -6$

Domain: \mathbb{R}

Range: $y < -6$

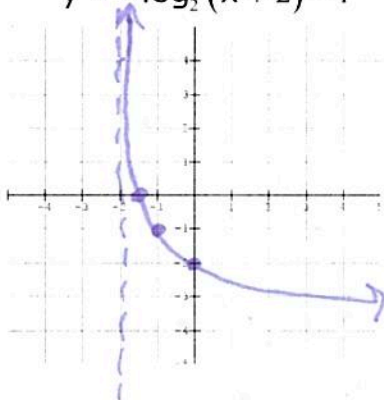
$$\begin{array}{r} 0, 1 \\ \times -4 \\ \hline 0, -4 \\ -6 \\ \hline 0, -10 \end{array}$$

Log Functions

Graph and Key Attributes

25) Graph and identify the key attributes. (Use the tables if they help you. Asymptote and critical point must be shown on the graph.)

$$y = -\log_2(x + 2) - 1$$



x	y
-1	1/2
0	1
1	2

Table for $y = (2)^x$

1/2	-1
1	0
2	1

Table for the Inverse

-2	x-1	-
-1.5	1	0
-1	0	-1
0	-1	-2
1		

Apply Transformations

Transformations: V. Reflection; left + 2; Down 1

Asymptote: $x = -2$

Domain: $x > -2$

Range: \mathbb{R}

26) Identify the key attributes given the following function:

$$y = -\frac{2}{3}\log_2(x + 2)$$

Transformations: V. Reflection
V. Compression
left + 2

Critical Point: $(-1, 0)$

Asymptote: $x = -2$

Domain: $x > -2$

Range: \mathbb{R}

$$\begin{array}{r} 1, 0 \\ -2 \\ \hline -1, 0 \end{array}$$

27) Apply the following transformations to the following function to write the new function and the new key attributes for the following functions:

Left 5, Up 3

$$y = \log(x)$$

New Equation: $y = \log(x + 5) + 3$

New Critical Point: $(-4, 3)$

New Asymptote: $x = -5$

New Domain: $x > -5$

New Range: \mathbb{R}

$$\begin{array}{r} 1, 0 \\ -5 + 3 \\ \hline -4, 3 \end{array}$$

Using Properties of Logs

28) Condense the following (make sure to simplify):

$$5\log(x) - \log(2)$$

$$\log x^5 - \log 2$$

$$\log \frac{x^5}{2}$$

$$\log_5(x) + \frac{1}{3}\log_5(8)$$

$$\log_5 x + \log_5 8^{1/3}$$

$$\log_5 x + \log_5 2$$

$$\log_5 2x$$

$$3\ln(2) - \ln(x) + \frac{2}{3}\ln(8)$$

$$\ln 2^3 - \ln x + \ln 8^{2/3}$$

$$\ln 8 - \ln x + \ln 4$$

$$\ln \frac{32}{x}$$

Solving Exponential/Log Equations (Common Base or by Converting)

Solve for the value of x:

29) $4^{3x} = \left(\frac{1}{8}\right)^{2x+1}$
 $2^{2(3x)} = 2^{-3(2x+1)}$
 $\frac{6x = -6x - 3}{+6x \quad +6x}$
 $12x = -3$
 $x = -\frac{1}{4}$

30) $\left(\frac{1}{6}\right)^x = 36$
 $6^{-x} = 6^2$
 $-x = 2$
 $x = -2$

31) $3(e)^{x-4} - 2 = 7$
 $\frac{+2 \quad +2}{3e^{x-4} = 9}$
 $e^{x-4} = 3$
 $\ln 3 = x - 4$
 $x = \ln(3) + 4$

32) $4(10)^{x+1} + 3 = 15$
 $\frac{-3 \quad -3}{4(10)^{x+1} = 12}$
 $\frac{4}{4} \frac{(10)^{x+1}}{4} = \frac{12}{4}$
 $10^{x+1} = 3$
 $\log 3 = x + 1$
 $x = \log(3) - 1$

33) $\log_7(2x^2) = \log_7(-11x + 21)$

$2x^2 = -11x + 21$
 $2x^2 + 11x - 21 = 0$
 $(2x-3)(x+7) = 0$
 $x = \frac{3}{2} \quad x = -7$

34) $2\log_7(4) - \log_7 x = \frac{2}{3}\log_7 64$
 $\log_7 4^2 - \log_7 x = \log_7 64^{2/3}$
 $\log_7 \frac{16}{x} = \log_7 16$
 $\frac{16}{x} = 16$
 $16 = 16x$
 $x = 1$

35) $\log_4(x^2 - 15x + 72) = 2$

$4^2 = x^2 - 15x + 72$
 $0 = x^2 - 15x + 56$
 $(x-8)(x-7)$
 $x = 8 \quad x = 7$

36) $2\log_4(3x) - 10 = -6$

~~$2\log_4(3x) = 4$~~
 ~~$\log_4(3x) = 2$~~
 ~~$4^2 = 3x$~~
 ~~$16 = \frac{3x}{3}$~~
 ~~$x = \frac{16}{3}$~~

Log/ Exponential Word Problems (Setting Up & Solving)

37) Is it better to invest your money at 4.9% interest compounded continuously or at 5.1% interest compounded monthly if you have \$21,000 to invest for 3 years?

$A = Pe^{rt}$

$A = 21,000e^{0.049(3)}$

$A = 24,325.43$

$y = 21,000\left(1 + \frac{0.051}{12}\right)^{12 \cdot 3}$

$y = 24,463.89$

Better option

38) If you have \$12,000 for principal earning and 4.8% interest after 4 years find the following if its compounded:

a.) Annually

$$12,000 \left(1 + \frac{.048}{1}\right)^{1 \cdot 4}$$

$$\boxed{14,475.26}$$

b.) Semi-annually

$$12,000 \left(1 + \frac{.048}{2}\right)^{2 \cdot 4}$$

$$\boxed{14,507.11}$$

c.) Quarterly

$$12,000 \left(1 + \frac{.048}{4}\right)^{4 \cdot 4}$$

$$\boxed{14,523.44}$$

d.) Monthly

$$12,000 \left(1 + \frac{.048}{12}\right)^{12 \cdot 4}$$

$$\boxed{14,534.78}$$

39) Nobelium-259 has a half-life of 58 minutes. How much remains of a 2-kg sample after 1 day?

↳ 1440 mins

$$1/2 \left(\frac{1}{2}\right)^{\frac{1440}{58}}$$

$$\boxed{6.72 \times 10^{-8}}$$

40) As a town gets smaller, the population of its high school decreases by 13% each year. The student body has 551 students now. In how many years will it have 390 students?

$$y = 551(1 - .13)^x$$

$$\frac{390}{551} = \frac{551(1 - .13)^x}{551}$$

$$\log_{.87} \left(\frac{390}{551}\right) = x$$

$$\boxed{x = 2.48}$$

41) Your parents bought a boat for \$60,000. The boat will depreciate by 9.3% each year. When will the boat be worth \$10,000?

$$\frac{10,000}{60,000} = \frac{60,000(1 - .093)^x}{60,000}$$

$$\log_{.907} \left(\frac{10,000}{60,000}\right) = x$$

$$\boxed{x = 18.36}$$

42) Mosquitoes are tripling in number each week. If there are currently 300 mosquitoes in your bug zapper in the back yard, when will there be 3500 mosquitoes?

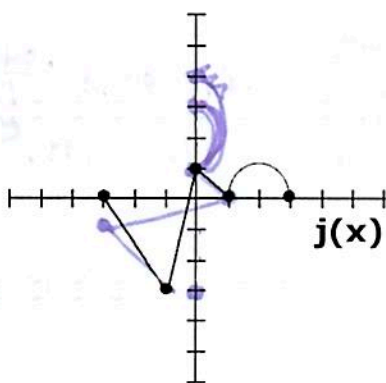
$$\frac{3500}{300} = \frac{300(3)^x}{300}$$

$$\log_3 \left(\frac{3500}{300}\right) = x$$

$$\boxed{x = 2.24}$$

Inverses

43) Graph the inverse of the function $j(x)$ on the same grid. (Hint: key points)



$j(x)$	x
-3	0
-1	-3
0	1
1	0
3	0

$j^{-1}(x)$	x
0	-3
-3	-1
1	0
0	1
0	3

44) Use the following functions to find the indicated compositions:

$$f(x) = x^2 - 5, \quad g(x) = 3x + 1, \quad h(x) = \frac{2}{x}$$

$$h(f(g(2)))$$

$$h(f(3(2)+1))$$

$$h(f(7))$$

$$h(7^2 - 5)$$

$$h(44)$$

$$\frac{2}{44} = \frac{1}{22}$$

$$(g \circ f)(-1) \quad g(f(-1))$$

$$g((-1)^2 - 5)$$

$$g(-4)$$

$$3(-4) + 1$$

$$-12 + 1 = -11$$

$$h(g(f(x)))$$

$$h(g(x^2 - 5))$$

$$h(3(x^2 - 5) + 1)$$

$$(3x^2 - 15 + 1)$$

$$h(3x^2 - 14)$$

$$\frac{2}{3x^2 - 14}$$

45) Use composition of functions to verify that the given two functions are inverses of each other:

$$f(x) = (x+3)^2 \quad \text{and} \quad g(x) = \sqrt{x} - 3$$

$$f(g(x)) = (\sqrt{x} - 3 + 3)^2$$

$$(\sqrt{x})^2 = \boxed{x} \checkmark$$

$$g(f(x)) = \sqrt{(x+3)^2} - 3$$

$$x+3 - 3 = \boxed{x} \checkmark$$

Yes

$$f(x) = (x-5)^3 \quad \text{and} \quad g(x) = \sqrt[3]{x+5}$$

$$f(g(x)) = (\sqrt[3]{x+5} - 5)^3$$

~~NO~~

NO

46) Find $f(x)^{-1}$ of the following functions:

$f(x) = 2x^2 + 3$, with domain $x \leq 0$ *negative!*

$$x = 2y^2 + 3$$

$$\frac{x-3}{2} = \frac{2y^2}{2}$$

$$y^2 = \frac{x-3}{2}$$

$$y = \sqrt{\frac{x-3}{2}}$$

$$f(x) = \sqrt[3]{x-8}$$

$$x = 3y-8$$

$$x+8 = 3y$$

$$y = \frac{x+8}{3}$$

$$f(x) = x^3 - 13$$

$$x = y^3 - 13$$

$$x+13 = y^3$$

$$y = \sqrt[3]{x+13}$$

$$f(x) = (3)^{x+2} + 1$$

$$x = 3^{y+2} + 1$$

$$x-1 = 3^{y+2}$$

$$\log_3(x-1) = y+2$$

$$\log_3(x-1) - 2 = y$$

$$y = \log_3(x-1) - 2$$

$$f(x) = e^x - 2$$

$$x = e^y - 2$$

$$x+2 = e^y$$

$$y = \ln(x+2)$$

$$f(x) = \log_3(x+2) - 1$$

$$x = \log_3(y+2) - 1$$

$$x+1 = \log_3(y+2)$$

$$3^{x+1} = y+2$$

$$y = 3^{x+1} - 2$$

$$f(x) = \log x - 6$$

$$x = \log y - 6$$

$$x+6 = \log y$$

$$10^{x+6} = y$$

$$y = 10^{x+6}$$

$$f(x) = \ln(5x)$$

$$x = \ln(5y)$$

$$e^x = 5y$$

$$y = \frac{1}{5}e^x \text{ or } y = \frac{e^x}{5}$$

Conics

47) Graph and identify the key attributes:

$$(x+4)^2 = -8(y-2)$$

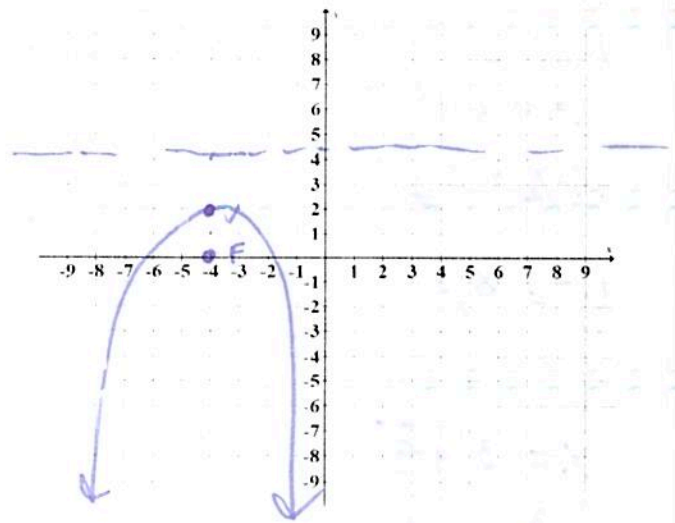
Vertex: $(-4, 2)$

$p = -2$

Focus: $(-4, 0)$

Directrix: $y = 4$

Axis: $x = -4$



48) Write the equation of a parabola with the following attributes:

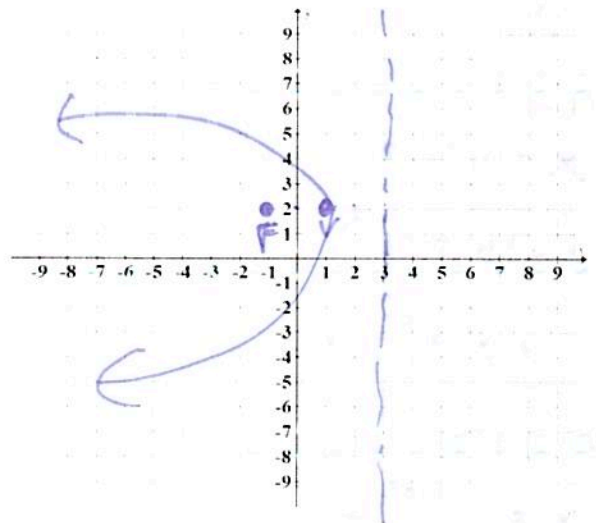
Directrix: $x = 3$ Focus: $(-1, 2)$

$V: (1, 2)$

$p: -2$

Equation: _____

$$(y-2)^2 = -8(x-1)$$



49) Graph and identify the key attributes:

$$(y+2)^2 = 4(x)$$

Vertex: $(0, -2)$

$p = 1$

Focus: $(1, -2)$

Directrix: $x = -1$

Axis: $y = -2$

