RULES FOR GRAPHING RATIONAL FUNCTIONS

Rational Function: a function that can be written as the ratio of two polynomials where the denominator is not equal to zero

$$f(x) = \frac{p(x)}{q(x)}$$

Asymptotes:

<table>
<tr><th>Horizontal Asymptotes (HA)</th><th>Vertical Asymptotes (VA)</th></tr>
<tr><td>Compare the degree of p(x) and q(x)</td><td rowspan="7">Roots of the denominator that do not cancel

(If they cancel, that means it has a removable discontinuity)</td></tr>
<tr><td>BOBO</td></tr>
<tr><td>If the degree of the denominator is greater, it’s y = 0</td></tr>
<tr><td>BOTN</td></tr>
<tr><td>If the degree of the numerator is bigger, no HA</td></tr>
<tr><td>COCO</td></tr>
<tr><td>If the degree of the numerator = the degree of the denominator,
the asymptote is the ratio of the leading coefficients.</td></tr>
</table>

Holes:

Any factor that appears in both the numerator and the denominator
will cancel.

A hole occurs when you set that factor equal to zero and solve for x.

To find the y value of the hole, plug the x value back into the
simplified equation.

In the example to the right, the hole occurs at (-2, 1).

$$y = \frac{\cancel{(x+2)}}{(x+3)\cancel{(x+2)}}$$

X+ 2 = 0

X = -2

Hole occurs when x = -2

Plug in -2 for x in the simplified equation.

$$y = \frac{1}{(-2+3)}$$

Y = 1

Domain: the domain of a rational function is all real numbers except for the x values of the vertical asymptotes and the x-coordinate of the hole.

Range: The range of a rational function is all real numbers except for the y values at horizontal asymptotes and the y-coordinate of the hole.