

A police car is dispatched to catch a bank robber, who is driving away from the scene of his latest crime. The robber is already 10 miles ahead of the police car, and the distance that he is traveling is represented by the equation $d = t + 10$, where t represents the time in minutes and d represents the distance traveled in miles. The distance that the police car is traveling is represented by the equation $d = 1.5t$.

a. Will the police car catch the robber? If so, when will this occur? Explain your reasoning.

$$d = t + 10$$

$$d = 1.5t$$

$$1.5t = t + 10$$

$$-t \quad -t$$

$$\frac{.5t}{.5} = \frac{10}{.5}$$

$$t = 20$$

in 20 minutes
they will
catch the
robber

$$d = 30 \text{ miles}$$

2. A cable television provider advertised three packages to new customers:

I. Two HD Sports channels, one HD Movie channel, and one HD Concert channel for \$20 a month. x y z

II. Three HD Sports channels and two HD Movie channels for \$30 a month.

III. One HD Movie channel and two HD Concert channels for \$18 a month.

Write and solve a system of equations using matrices to find out the price for each channel. Be sure to define your variables.

$$2x + y + z = 20$$

$$3x + 2y + 0z = 30$$

$$0x + y + 2z = 18$$

$$\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 0 \\ 0 & 1 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 20 \\ 30 \\ 18 \end{bmatrix}$$

$$x = \$2.80$$

$$y = \$10.80$$

$$z = \$3.60$$

3. For one day, the movie theater decided to roll back the prices to celebrate its grand opening.

Children: \$2

Adults: \$4

Senior Citizens: \$3

By the end of the day, the movie theater had sold 110 tickets and collected \$344 from ticket sales. On this day, the number of adult tickets sold was equal to the number of children tickets and senior citizen tickets combined.

Calculate the number of children, adult, and senior citizen tickets were sold. Be sure to define your variables, formulate a system of equations, and use matrices to solve the system.

$$2c + 4a + 3s = \$344$$

$$c + a + s = 110$$

$$a = c + s$$

$$-c + a - s = 0$$

$$\begin{bmatrix} 2 & 4 & 3 \\ 1 & 1 & 1 \\ -1 & 1 & -1 \end{bmatrix}^{-1} \begin{bmatrix} 344 \\ 110 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} c = 41 \\ a = 55 \end{bmatrix} \quad \begin{bmatrix} s = 14 \end{bmatrix}$$

For #4-5, solve each system of two equations in two variables by hand using substitution.

$$4. \begin{cases} y = x^2 + x - 8 \\ x + y = 7 \end{cases}$$

$$y = 7 - x$$

$$7 - x = x^2 + x - 8$$

$$0 = x^2 + 2x - 15$$

$$(x + 5)(x - 3)$$

$$x = -5$$

$$x = 3$$

$$\begin{array}{r} -5 + y = 7 \\ +5 \quad +5 \\ \hline y = 12 \end{array}$$

$$\begin{array}{r} 3 + y = 7 \\ -3 \quad -3 \\ \hline y = 4 \end{array}$$

$$5. \begin{cases} y = x^2 - 6x \\ -6x + y = -32 \end{cases}$$

$$+6x \quad +6x$$

$$y = -32 + 6x$$

$$-32 + 6x = x^2 - 6x$$

$$0 = x^2 - 12x + 32$$

$$0 = (x - 8)(x - 4)$$

$$x = 8$$

$$x = 4$$

$$y = 8^2 - 6(8)$$

$$y = 16$$

$$y = 4^2 - 6(4)$$

$$y = -8$$

For all problems, define variables, write the system of equations and solve for all variables by using each method (substitution, elimination, graphing, and matrix) at least once.

- Chase and Sara went to the candy store. Chase bought 5 pieces of fudge and 3 pieces of bubble gum for a total of \$5.70. Sara bought 2 pieces of fudge and 10 pieces of bubble gum for a total of \$3.60. Which system of equations could be used to determine the cost of 1 piece of fudge, f , and 1 piece of bubble gum, g ?
- Some students want to order shirts with their school logo. One company charges \$9.65 per shirt plus a setup fee of \$43. Another company charges \$8.40 per shirt plus a \$58 fee. For what number of shirts would the cost be the same?
- The concession stand at the basketball game sells candy bars, drinks, and bags of chips. On Thursday, they sold 50 candy bars, 125 drinks, and 75 bags of chips and made a \$268.75. On Friday, they sold 62 candy bars, 150 drinks, and 72 bags of chips and made \$306. On Saturday, they ran out of bags of chips, so they sold 101 candy bars and 125 drinks for a total \$232. What is the price of each item?
- Ms. Kitts works at a music store. Last week she sold 6 more than 3 times the number of CDs that she sold this week. Ms. Kitts sold a total of 110 CDs over the 2 weeks. Which system of equations can be used to find l , the number of CDs she sold last week, and t , the number of CDs she sold this week?

Solve the following systems first by using a calculator (matrices or graphing) and then without a calculator (substitution and elimination).

$$10. \begin{cases} x+5y-z=16 \\ 3x-3y+2z=12 \\ 2x+4y+z=20 \end{cases}$$

$$11. \begin{cases} -2x+y+3z=-8 \\ 3x+4y-2z=9 \\ x+2y+z=4 \end{cases}$$

$$12. \begin{cases} 9x+5y-z=-11 \\ 6x+4y+2z=2 \\ 2x-2y+4z=4 \end{cases}$$

$$13. \begin{cases} 11x-7y=-14 \\ x=-4+2y \end{cases}$$

$$14. \begin{cases} 2(2x-3y=6) \\ -4x+6y=6 \end{cases}$$

$4x-6y=12$
 $0=18$
NO SOLUTION

$$15. \begin{cases} y=3x-10 \\ y=3x+11 \end{cases}$$

NO SOLUTION

$$16. \begin{cases} 3x-2y=-5 \\ 4x+5y=47 \end{cases}$$

$$17. \begin{cases} y=\frac{1}{2}x-6 \\ x-2y=12 \end{cases}$$

infinite solutions

$-x \quad -x$
 $-2y=12-x$
 $\frac{-2y}{-2}=\frac{12-x}{-2}$
 $y=-6+\frac{1}{2}x$



6 f = fudge
g = bubble gum

$$\begin{aligned} 5f + 3g &= 5.70 \\ 2f + 10g &= 3.60 \end{aligned}$$

Solve by:
- matrix
- elimination

Matrix: $\begin{bmatrix} 5 & 3 \\ 2 & 10 \end{bmatrix}^{-1} \begin{bmatrix} 5.70 \\ 3.60 \end{bmatrix} = \begin{bmatrix} \$1.05 \\ \$0.15 \end{bmatrix}$ fudge
bubble gum

2 X = # shirts
7 y = cost

$$\begin{aligned} y &= 9.65x + 43 \\ y &= 8.40x + 58 \end{aligned}$$

Solve by:
- graphing
- substitution

$$\begin{array}{r|l} 9.65x + 43 & = 8.40x + 58 \\ -8.40x - 43 & \quad \quad -8.40x - 43 \\ \hline 1.25x & = 15 \end{array}$$

$$\frac{1.25x}{1.25} = \frac{15}{1.25}$$

$$= \frac{15}{1.25}$$

$$x = 12$$

12 shirts

$$y = 9.65(12) + 43 \rightarrow y = \$158.80 \text{ cost}$$

3 c = \$ candy bar
d = \$ drink
b = \$ bag of chips
solve by matrix!

$$\begin{aligned} 50c + 125d + 75b &= 268.75 \\ 62c + 150d + 72b &= 306 \\ 101c + 125d &= 232 \end{aligned}$$

$$\begin{bmatrix} 50 & 125 & 75 \\ 62 & 150 & 72 \\ 101 & 125 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 268.75 \\ 306 \\ 232 \end{bmatrix} = \begin{bmatrix} \$0.75 \\ \$1.25 \\ \$1 \end{bmatrix}$$
 candy bar
drink
chips

4 l = last week
t = this week

$$\begin{aligned} l &= 3t + 6 \\ l + t &= 110 \end{aligned}$$

Solve by:
- ~~graphing~~
substitution

$$3t + 6 + t = 110$$

$$4t + 6 = 110$$

$$\frac{4t}{4} = \frac{104}{4}$$

$$t = 26, l = 84$$

$$\begin{aligned} (16) \quad & ① \quad x + 5y - z = 16 \\ & ② \quad 3x - 3y + 2z = 12 \\ & ③ \quad 2x + 4y + z = 20 \end{aligned}$$

cancel z's!

$$\begin{aligned} x + 5y - \cancel{z} &= 16 \\ + 2x + 4y + \cancel{z} &= 20 \\ \hline 3x + 9y &= 36 \end{aligned}$$

$$\begin{aligned} 2(x + 5y - z) &= 16 \\ 3x - 3y + 2z &= 12 \end{aligned}$$

$$\begin{aligned} 2x + 10y - \cancel{2z} &= 32 \\ + 3x - 3y + \cancel{2z} &= 12 \\ \hline 5x + 7y &= 44 \end{aligned}$$

cancel x's!

$$\begin{aligned} 5(3x + 9y) &= 36 \\ -3(5x + 7y) &= 44 \end{aligned}$$

$$\begin{aligned} 15x + 45y &= 180 \\ -15x + 21y &= -132 \\ \hline 24y &= 48 \\ y &= 2 \end{aligned}$$

solve for x!

$$\begin{aligned} 3x + 9(2) &= 36 \\ 3x + 18 &= 36 \\ 3x &= 18 \\ x &= 6 \end{aligned}$$

~~solve for y!~~

solve for z!

$$\begin{aligned} 6 + 5(2) - z &= 16 \\ 16 - z &= 16 \\ -z &= 0, \quad z = 0 \end{aligned}$$

Sol: (6, 2, 0)

$$\begin{aligned} (17) \quad & ① \quad -2x + y + 3z = -8 \\ & ② \quad 3x + 4y - 2z = 9 \\ & ③ \quad x + 2y + z = 4 \end{aligned}$$

cancel y's!

$$\begin{aligned} -2x + y + 3z &= -8 \\ x + 2y + z &= 4 \end{aligned}$$

$$\begin{aligned} 4x - 8y - 6z &= -16 \\ + x + 2y + z &= 4 \\ \hline 5x - 5z &= 20 \end{aligned}$$

solve for x's!

$$\begin{aligned} 5x - 5z &= 20 \\ -5(x - 4z) &= -5 \end{aligned}$$

$$\begin{aligned} 3x + 4y - 2z &= 9 \\ -2x + 2y + z &= 4 \end{aligned}$$

$$\begin{aligned} 3x + 4y - 2z &= 9 \\ + -2x - 4y + z &= -8 \\ \hline x - 4z &= 1 \end{aligned}$$

$$\begin{aligned} 5x - 5z &= 20 \\ -5x + 20z &= -5 \\ \hline 15z &= 15 \\ z &= 1 \end{aligned}$$

$$\begin{aligned} x - 4(1) &= 1 \\ x - 4 &= 1 \\ x &= 5 \end{aligned}$$

solve for y!

$$\begin{aligned} 5 + 2y + 1 &= 4 \\ 2y + 6 &= 4 \end{aligned}$$

$$2y = -2 \quad y = -1$$

Sol: (5, -1, 1)

$$\textcircled{12} \begin{cases} 9x + 5y - z = -11 \\ 6x + 4y + 2z = 2 \\ 2x - 2y + 4z = 4 \end{cases}$$

solve for z's!

$$\begin{array}{r} 2(9x + 5y - z = -11) \\ 6x + 4y + 2z = 2 \end{array} \quad \begin{array}{r} > 18x + 10y - 2z = -22 \\ \oplus 6x + 4y + 2z = 2 \\ \hline 24x + 14y = -20 \end{array}$$

$$\begin{array}{r} -2(6x + 4y + 2z = 2) \\ 2x - 2y + 4z = 4 \end{array} \quad \begin{array}{r} > -12x - 8y - 4z = -4 \\ \oplus 2x - 2y + 4z = 4 \\ \hline -10x - 10y = 0 \end{array}$$

$$2(-2) - 2(2) + 4z = 4$$

$$-4 - 4 + 4z = 4$$

$$4z = 12 \quad \textcircled{z=3}$$

$$\boxed{\text{sol: } (-2, 2, 3)}$$

$$\begin{array}{r} 5(24x + 14y = -20) \\ 12(-10x - 10y = 0) \\ \hline \end{array}$$

$$\begin{array}{r} \oplus 120x + 70y = -100 \\ -120x - 120y = 0 \\ \hline -50y = -100 \\ \textcircled{y=2} \end{array}$$

$$-10x - 10(z) = 0$$

$$-10x - 20 = 0$$

$$-10x = 20$$

$$\textcircled{x=-2}$$

$$\textcircled{13} \begin{cases} 11x - 7y = -14 \\ x = -4 + 2y \end{cases}$$

$$\begin{array}{r} \rightarrow 11(-4 + 2y) - 7y = -14 \\ -44 + 22y - 7y = -14 \\ -44 + 15y = -14 \\ \hline +44 \quad \quad \quad | +44 \\ 15y = 30 \end{array}$$

$$15y = 30$$

$$\textcircled{y=2}$$

$$x = -4 + 2(z)$$

$$\textcircled{x=0}$$

$$\boxed{\text{sol: } (0, 2)}$$

$$\textcircled{4} \begin{cases} 2x - 3y = 6 \\ -4x + 6y = 6 \end{cases}$$

$$4x - 6y = 12$$

$$-4x + 6y = 6$$

$$0 = 18 \quad \text{False!}$$

$$\boxed{\text{No Solution}}$$

$$\begin{array}{l} (15) \ y = 3x - 10 \\ \quad y = 3x + 11 \end{array} >$$

↑
Parallel Lines!

$$\begin{array}{r} 3x - 10 = 3x + 11 \\ -3x \quad \quad -3x \\ \hline \end{array}$$

$$-10 = 11 \quad \text{False!}$$

No Solution

$$\begin{array}{l} (16) \ 3x - 2y = -5 \\ \quad 3(4x + 5y = 47) \end{array} >$$

$$\begin{array}{r} 3x - 2(7) = -5 \\ \quad +14 \quad \quad +14 \\ \hline 3x = 9 \\ \quad x = 3 \end{array}$$

$$\begin{array}{r} -12x + 8y = 20 \\ \oplus 12x + 15y = 141 \\ \hline \end{array}$$

$$23y = 161 \\ y = 7$$

Sol: (3, 7)

$$\begin{array}{l} (17) \ y = \frac{1}{2}x - 6 \\ \quad x - 2y = 12 \end{array} >$$

$$\begin{array}{l} x - 2\left(\frac{1}{2}x - 6\right) = 12 \\ x - x + 12 = 12 \end{array}$$

$$12 = 12 \quad \text{True}$$

Infinitely Many Solutions

