PAP Algebra **Name:** key

Chapter 12 Test Review

Solving Exponential/Log Equations (Common Base or by Converting)

1) $2^x = 16$

$2^x = 2^4$

$\boxed{x = 4}$

2) $4^{3x} = \left(\frac{1}{8}\right)^{2x+1}$

$2^{2(3x)} = 2^{-3(2x+1)}$

$6x = -6x - 3$

$+6x \quad +6x$

$\frac{12x}{12} = \frac{-3}{12}$ $\boxed{x = -1/4}$

3) $3(e)^x + 3 = 9$

$-3 \quad -3$

$\frac{3e^x}{3} = \frac{6}{3}$

$e^x = 2$

$\boxed{\ln 2 = x}$ or $\boxed{.693}$

4) $4(e)^{x+5} - 8 = 12$

$+8 \quad +8$

$\frac{4e^{x+5}}{4} = \frac{20}{4}$

$e^{x+5} = 5$

$\ln 5 = x + 5$

$\boxed{x = \ln(5) - 5 \text{ or } -3.39}$

5) $\left(\frac{1}{3}\right)^x = 27$

$3^{-1x} = 3^3$

$-x = 3$

$\boxed{x = -3}$

6) $\log_2(x^2 + 3x - 10) = 3$

$2^3 = x^2 + 3x - 10$

$8 = x^2 + 3x - 10$

$0 = x^2 + 3x - 18$

$(x + 6)(x - 3)$

~~$x = -6$~~

$\boxed{x = 3}$

7) $\frac{3\ln(x+5)}{3} = \frac{21}{3}$

$\ln(x+5) = 7$

$e^7 = x + 5$

$\boxed{x = e^7 - 5}$ or $\boxed{x = 1091.633}$

8) $2\log_7(4) - \log_7 x = \frac{2}{3}\log_7 8$

$\log_7 4^2 - \log_7 x = \log_7 8^{2/3}$

$\log_7 \frac{16}{x} = \log_7 4$

$\frac{16}{x} = \frac{4}{1}$

$\frac{16}{4} = \frac{4x}{4}$

$\boxed{x = 4}$

9) $14 + \log_7(x) = 16$

$-14 \quad -14$

$\log_7(x) = 2$

$7^2 = x$

$\boxed{x = 49}$

10) $\log_6(3x) - 10 = -8$

$+10 \quad +10$

$\log_6(3x) = 2$

$6^2 = 3x$

$\frac{36}{3} = \frac{3x}{3}$

$\boxed{x = 12}$

11) $\ln(x) = 13 - \ln(x^2)$

$\ln x + \ln x^2 = 13$

$\ln x^3 = 13$

$e^{13} = x^3$

$\sqrt[3]{e^{13}} = x$

$\boxed{x = 76.198}$

12) $\log_3(x+8) - \log_3(x-4) = 2$

$\log_3 \frac{x+8}{x-4} = 2$

$3^2 = \frac{x+8}{x-4}$

$\frac{9}{1} = \frac{x+8}{x-4} \rightarrow 9(x-4) = x+8$

$9x - 36 = x + 8$

$-x \quad -x$

$8x - 36 = 8$

$+36 \quad +36$

$8x = 44$

$\boxed{x = 5.5}$

13) $\log_4(4x) = 3 - \log_4(2x)$

$\log_4(4x) + \log_4(2x) = 3$

$\log_4(8x^2) = 3$

$4^3 = 8x^2$

$\frac{64}{8} = \frac{8x^2}{8}$

$\sqrt{x^2} = \sqrt{8}$

$\boxed{x = 2\sqrt{2} \text{ or } 2.83}$

14) $2\log_3(x) - \log_3(2) = 3\log_3(4)$

$\log_3(x^2) - \log_3(2) = \log_3(4)^3$

$\log_3 \frac{x^2}{2} = \log_3 64$

$\frac{x^2}{2} = \frac{64}{1}$

$\sqrt{x^2} = \sqrt{128}$

$\boxed{x = 11.313}$

Word Problems (Setting Up & Solving)

15) Is it better to invest your money at 5.5% interest compounded continuously or at 5.8% interest compounded monthly if you have $12,000 to invest for 4 years?

$Y = 12{,}000e^{.055x}$

$Y = 12{,}000e^{.055(4)}$

$\boxed{Y = 14{,}952.92}$

$Y = 12{,}000\left(1 + \frac{.058}{12}\right)^{12x}$

$12{,}000\left(1 + \frac{.058}{12}\right)^{12(4)}$

$\boxed{Y = 15{,}124.98}$

Better option

16) $12,000 principal earning 4.8% interest after 4 years

a.) Annually

$12{,}000\left(1+\frac{.048}{1}\right)^{1x}$

$12{,}000\left(1+\frac{.048}{1}\right)^{1\cdot 4}$

14478.26

b.) Semi-annually

$12000\left(1+\frac{.048}{2}\right)^{2x}$

$12000\left(1+\frac{.048}{2}\right)^{2(4)}$

14507.109

c.) Quarterly

$12000\left(1+\frac{.048}{4}\right)^{4x}$

$12000\left(1+\frac{.048}{4}\right)^{4(4)}$

14523.438

d.) Monthly

$12000\left(1+\frac{.048}{12}\right)^{12x}$

$12000\left(1+\frac{.048}{12}\right)^{12\cdot 4}$

14534.478

17) If you have an account that has an interest rate of 1.9% compounded monthly, how long will it take for your money to triple?

Initial = 100

$\frac{300}{100}=\frac{100}{100}\left(1+\frac{.019}{12}\right)^{12x}$

$3=\left(1+\frac{.019}{12}\right)^{12x}$

$\frac{\log\left(1+\frac{.019}{12}\right)^{12x}}{12}=\frac{\log 3}{12}$

$x=57.87$

18) Mosquitoes are tripling in number each week. If there are currently 300 mosquitoes in your bug zapper in the back yard, when will there be 2000 mosquitoes?

$\frac{2{,}000}{300}=\frac{300(3)^x}{300}$

least Rounded

$6\frac{2}{3}=3^x$

$\log_3\left(6\frac{2}{3}\right)=x$ or $\log_3\left(\frac{2000}{300}\right)=x$

$x=1.731$

19) As a town gets smaller, the population of its high school decreases by 12% each year. The student body has 538 students now. In how many years will it have 390 students?

$y=538(1-.12)^x$

$\frac{390}{538}=\frac{538(.88)^x}{538}$

$\log_{.88}\left(\frac{390}{538}\right)=x$

$x=2.52$

20) The world population in 2000 was approx. 6.08 billion. The annual rate of increase was about 1.26%. If the world population continues to grow at this rate, when will the population reach 9 billion?

→ e

.0126

$9=6.108e^{.0126x}$

$\frac{9}{6.08}=\frac{6.08e^{.0126x}}{6.08}$

$\frac{\ln\frac{9}{6.08}}{.0126}=\frac{.0126x}{.0126}$

$x=31$ years

21) Nobelium-259 has a half-life of 58 minutes. How much remains of a 1 kg sample after 1 day?

↳ 1440

$y = 1(1/2)^{\frac{1440}{58}}$

$y = 3.359 \times 10^{-8}$

22) Dubnium-262 has a half-life of 34 seconds. How many grams did we begin with if, after 5 minutes, we are left with only 1 gram?

↳ 300

$\frac{1}{(1/2)^{\frac{300}{34}}} = \frac{x(1/2)^{\frac{300}{34}}}{(1/2)^{\frac{300}{34}}}$

$x = 453.1$

23) Your parents bought a boat for \$60,000. The boat will depreciate by 9.3% each year. When will the boat be worth \$10,000?

1 − r

$\frac{10{,}000}{60{,}000} = \frac{60{,}000(1-.093)^x}{60{,}000}$

$\frac{1}{6} = (1-.093)^x$

$\log_{.907} 1/6 = x$

$x = 18.36$

24) If a population of 175 red-spotted toads doubles every 2 years, how many toads can you expect to find in 10 years?

$y = 175(2)^{\frac{x}{2}}$

$y = 175(2)^{\frac{10}{2}}$

$y = 5{,}600$

25) A student wants to have \$8000 for college 5 years from now. How much should she put into an account that earns 5.2% annual interest compounded continuously?

$\frac{8{,}000}{e^{.052(5)}} = \frac{xe^{.052(5)}}{e^{.052(5)}}$

$6{,}168.41 = x$