PAP Algebra 2
Notes: Complex Numbers

Name: Key
Period: ______ Date: ______

Complex

REAL NUMBERS
Rational + Irrational #'s Represented by "a"

Rational
any # that can be expressed as a fraction (finite decimals)
$-3/2, -1, -1/2, 0, \frac{1}{2}, 1, \frac{3}{2}$

Integers
Positive / negative & 0
$-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6$

Whole
Positive whole #'s including 0
$0, 1, 2, 3 \ldots$

Natural/Counting
Positive whole #s
$1, 2, 3, 4, 5$

Irrational
#'s that cannot be written as a fraction
π, $\sqrt{5}$
e, $\sqrt{2}$
$.1001000l\ldots$

IMAGINARY NUMBERS
The square root of a negative #
Represented by "bi"
$i = \sqrt{-1}$
$\sqrt{-4} \Rightarrow 2i$
$\sqrt{-3} \to 3i$
πi $4i$
ei $\frac{1}{2}i$

Consider the equation $x^2 = -1$. Is there a real number solution to this equation? No!

$x = \sqrt{-1}$

The imaginary number i is a number such that $i^2 = -1$. Because no real number exists such that its square is equal to a negative number, the number i is not a part of the real number system.

If $i^2 = -1$, then what is the value of i?

Examples: Rewrite each expression using i.

1. $\sqrt{64} - \sqrt{-63}$

$8 - i\sqrt{63}$

$\sqrt{9}\ \sqrt{7}$

$\boxed{8 - 3i\sqrt{7}}$

2. $\sqrt{-13} + 10$

$\boxed{10 + i\sqrt{13}}$

3. $\dfrac{1-\sqrt{-44}}{2}$

$\dfrac{1 - i\sqrt{44}}{2}$

$\sqrt{44}$

$\sqrt{4}\ \sqrt{11}$

$\dfrac{1 - i\sqrt{44}}{2}$

$\boxed{\dfrac{1 - 2i\sqrt{11}}{2}}$

or

$\boxed{\frac{1}{2} - i\sqrt{11}}$

The **set of complex numbers** is the set of all numbers written in the form $a \pm bi$, where a and b are real numbers. The a term is called the **real part of a complex number**, and the bi term is called the **imaginary part of a complex number**.

Examples: Write the following as a complex number. Identify the a and b.

4. i

$a = 0$ $\quad 0 \pm i$
$b = 1$

5. 3

$3 \pm 0i$
$a = 3$
$b = 0$

6. $\pi + 3.2i$

$a = \pi$
$b = 3.2$

When operating with complex numbers involving i, combine like terms by treating i as a variable (even though it is a constant.)

Simplify each expression.

7. $(3 + 2i) - (1 - 6i)$

$\boxed{2+8i}$

8. $4i + 3 - 6 + i - 1$

$\boxed{-4+5i}$

9. $7i + 4 - 2i - 10$

$\boxed{-6+5i}$

Determine each product.

10. $5i(3 - 2i)$

$15i - 10i^2$

$15i - 10(-1)$

$\boxed{10+15i}$

11. $(5 + 3i)(2 - 3i)$

$10 - 15i + 6i - 9i^2$

$10 - 9i - 9(-1)$

$\boxed{19-9i}$

12. $(1 - 3i)(1 + 3i)$

$1 + 3i - 3i - 9i^2$

$1 + 9 = \boxed{10}$

What do you notice about #12?

No Imaginary

Complex conjugates are pairs of numbers of the form $a + bi$ and $a - bi$. The product of a pair of complex conjugates is always a real number and equal to $a^2 + b^2$. Division of complex numbers requires the use of complex conjugates because imaginary numbers are not allowed to remain in the denominator. You can rewrite the division of complex numbers by multiplying both the numerator and denominator by the conjugate of the denominator.

Examples: Rewrite each quotient without a complex number in the denominator.

13. $\dfrac{2-i}{3+2i}\dfrac{(3-2i)}{(3-2i)}$

$\dfrac{(2-i)(3-2i)}{9+4} = \dfrac{6-4i-3i+2i^2}{13}$

$\boxed{\dfrac{4-7i}{13}}$

14. $\dfrac{3-4i}{2-3i}\dfrac{(2+3i)}{(2+3i)}$ $\dfrac{6+9i-8i-12i^2}{4+9}$

$\boxed{\dfrac{18+i}{13}}$

15. $\dfrac{5+2i}{1+i}\dfrac{(1-i)}{(1-i)} = \dfrac{5-5i+2i+2i^2}{1+1}$

$\boxed{\dfrac{7-3i}{2}}$

16. $\dfrac{20-5i}{2-4i}\dfrac{(2+4i)}{(2+4i)}$ $\dfrac{40+80i-10i-20i^2}{4+16}$

$\dfrac{60+70i}{20} = \boxed{\dfrac{6+7i}{2}}$

Explain: Writing Quadratic Equations

1. You have been chosen to work on a new project for Frisco ISD to install their own cell phone service for students and staff. You have plotted out the region that needs to be covered by service and determined that the region is quadratic and has roots at -3 and 5 on your graph with an a value of -2.

   a) What would the factors of your equation need to be in order to have roots at -3 and 5?

   ___$x+3$___ and ___$x-5$___

   b) What then is the equation, in factored form?

   $y=-2(x+3)(x-5)$

   c) Multiply out your equation and give the equation in standard form.

   $y=-2(x^2-5x+3x-15)$

   $y=-2(x^2-2x-15)$

   $\boxed{y=-2x^2+4x+30}$

2. Now, you need a quadratic that has roots of $\frac{1}{2}$ and $-\frac{2}{3}$

   a) What would the factors of this new equation be?

   ___$x-1/2$___ and ___$x+2/3$___

   b) When writing an equation you can't have fractions in the factors. To fix that problem you multiply each factor by the common denominator of that factor. (Example: $\left(x-\frac{2}{5}\right)$ would be multiplied by 5 to yield (5x – 2). Notice that if you set 5x -2 equal to 0 and solve you get $\frac{2}{5}$)

   What is the equation of your quadratic in both factored and standard forms?

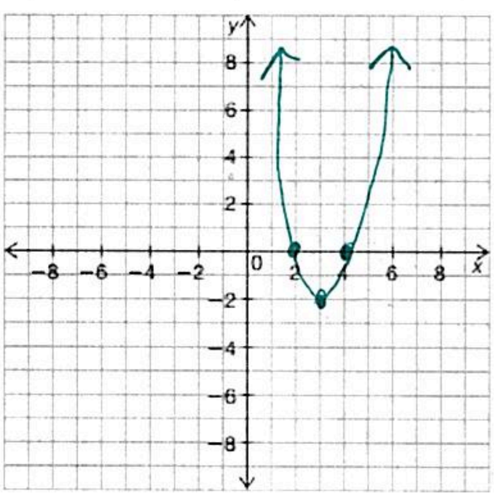
   $y=(2x-1)(3x+2)$

   $y=6x^2+4x-3x-2$

   $\boxed{y=6x^2+x-2}$

   ★ Any "A" value would have the same Roots! ★

3. Given, the vertex is (3, -2) and one of the two x-intercepts is (4,0). Sketch and determine the equation of the parabola in vertex form. **You must find the "a" value!**



$y=a(x-h)^2+k$

$y=a(x-3)^2-2$

$0=a(4-3)^2-2$

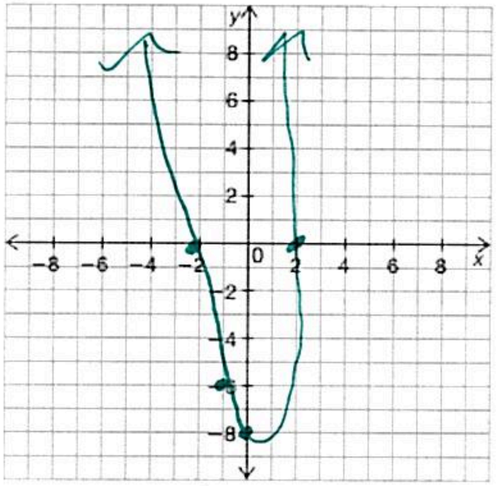
$0=a(1)^2-2$

$0=a-2$

$\boxed{a=2}$

$\boxed{y=2(x-3)^2-2}$

4. Given, the roots of the quadratic are (2,0) and (-2,0) and (-1,-6) is another point on the parabola. Sketch and determine the equation of the parabola in standard form.



$y = a(x-2)(x+2)$

$-6 = a(-1-2)(-1+2)$

$-6 = a(-3)(1)$

$\frac{-6}{-3} = \frac{-3a}{-3}$

$\boxed{a = 2}$

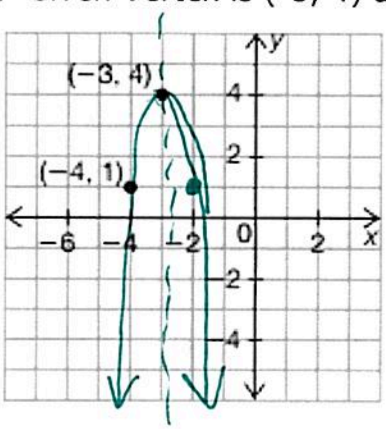
$y = 2(x-2)(x+2)$

$y = 2[x^2+2x-2x-4]$

$y = 2(x^2-4)$

$\boxed{y = 2x^2-8}$

5. Given Vertex is (-3, 4) and point (-4, 1) Write an equation of the parabola.



$y = a(x+3)^2+4$

$1 = a(-4+3)^2+4$

$1 = a(-1)^2+4$

$1 = a+4$

$-4 \quad -4$

$\boxed{a = -3}$

$\boxed{y = -3(x+3)^2+4}$

6. Given standard form $y = -3x^2 - 18x - 23$, determine vertex form.

(Hint: you can complete the square, or use the formula $x = \frac{-b}{2a}$ to find the x-value of the vertex.)

$y = -3x^2-18x-23$

$y+23 = -3x^2-18x$

$y+23+\underline{-27} = -3(x^2+6+\underline{9})$

$y-4 = -3(x+3)^2$

$\boxed{y = -3(x+3)^2+4}$

V: (-3,4)

OR

$a = -3$

$b = -18$

$c = -23$

$\frac{-(-18)}{2(-3)} = \frac{18}{-6} = \boxed{-3}$

$y = -3(-3)^2-18(-3)-23$

$y = -3(9)+54-23$

$y = -27+31$

$\boxed{y = 4}$

V: (-3,4)

$y = ax^2 + bx + c$

7. Create a system of equations and use algebra to create a quadratic equation with points (-1,5), (0,3), and (3,9).

$5 = a(-1)^2 + b(-1) + c \rightarrow 5 = a - b + c$

$3 = a(0)^2 + b(0) + c \longrightarrow 3 = c$

$9 = a(3)^2 + b(3) + c \rightarrow 9 = 9a + 3b + c$

$5 = a - b + 3$
$-3 \qquad -3$
$2 = a - b$

$9 = 9a + 3b + 3$
$6 = 9a + 3b$

$3(2 = a - b)$
$6 = 3a - 3b$
$6 = 9a + 3b$
$12 = 12a$
$\boxed{a = 1}$

$2 = 1 - b$
$-1 \quad -1$
$-b = 1$
$\boxed{b = -1}$

$\boxed{y = x^2 - x + 3}$

8. Create a system of equations and use algebra to create a quadratic equation with points (-4,12), (0,-16), and (-2,-14).

$12 = a(-4)^2 + b(-4) + c \rightarrow 12 = 16a - 4b + c$

$-16 = a(0)^2 + b(0) + c \rightarrow -16 = c$

$-14 = a(-2)^2 + b(-2) + c \rightarrow -14 = 4a - 2b + c$

$12 = 16a - 4b - 16$
$+16 \qquad +16$
$28 = 16a - 4b$

$-14 = 4a - 2b - 16$
$+16 \qquad +16$
$2 = 4a - 2b$

$-2(2 = 4a - 2b)$
$-4 = -8a + \cancel{4b}$
$28 = 16a - \cancel{4b}$
$\frac{24}{8} = \frac{8a}{8}$
$\boxed{a = 3}$

$2 = 4(3) - 2b$
$2 = 12 - 2b$
$-12 \; -12$
$-10 = -2b$
$\boxed{b = 5}$

$\boxed{y = 3x^2 + 5x - 16}$