

Sec. 5.5 Notes – Writing Polynomial Equations

Sometimes, we need to write quadratic equations given imaginary/complex roots.

1. For example, given $\pm 4i$ as your roots, create a polynomial in standard form.

a) First, write out the factors that you would use to obtain the complex solution.

$$(x+4i)(x-4i)$$

b) Next, multiply the factors.

	$x + 4i$	
x	x^2	$4xi$
$-4i$	$-4xi$	$-16i^2$

$$x^2 - 16i^2 \quad i^2 = -1$$

$x^2 + 16$

c) Write your polynomial function in standard form.

$x^2 + 16$

2. Let's try one more: Write the quadratic with roots at $\pm 3i$ and $\frac{5}{2}$

$$(x+3i)(x-3i)(2x-5)$$

	$x + 3i$	
x	x^2	$3xi$
$-3i$	$-3xi$	$-9i^2$

$$x^2 - 9i^2$$

$x^2 + 9$

	$x^2 + 9$	
$2x$	$2x^3$	$18x$
-5	$-5x^2$	-45

$$x = \frac{5}{2}$$

$$x - \frac{5}{2} = 0$$

$$2x - 5 = 0$$

$2x^3 - 5x^2 + 18x - 45$

3. Now, for the final type. What if your roots are $1 \pm 5i$?

- a) Write out the factors that you would use to obtain the complex solution. Remember to change signs of everything in your root when you put it into a factor.

$$x = 1 + 5i$$

$$x = 1 - 5i$$

$$(x - 1 - 5i)(x - 1 + 5i) = 0$$

- b) Using the box method, multiply the factors. Notice since you have 3 terms in each factor, the box is now a 3 x 3 box.

	x	-1	$-5i$	
x	x^2	$-x$	$-5xi$	$x^2 - 2x + 1 - 25i^2$ $x^2 - 2x + 1 + 25$ $x^2 - 2x + 26$
-1	$-x$	1	$5i$	
$+5i$	$5xi$	$-5i$	$-25i^2$	

- c) Write your polynomial function in standard form from your boxes. Note, you cannot leave i^2

$$x^2 - 2x + 26$$

4. Now, let's practice with $2 \pm 3i$

- a) What are the factors?

$$(x - 2 - 3i)(x - 2 + 3i)$$

- b) Multiply out the factors using a box:

	x	-2	$-3i$	
x	x^2	$-2x$	$-3xi$	$x^2 - 4x + 4 - 9i^2$ $x^2 - 4x + 4 + 9$
-2	$-2x$	4	$6i$	
$+3i$	$3xi$	$-6i$	$-9i^2$	

- c) What is the equation of your quadratic in standard form?

$$x^2 - 4x + 13$$

Write the equation in standard form of a polynomial with the following solutions/roots/zeros.

1. $x = -3$ and $x = 15$

$$(x+3)(x-15) = 0$$

$$x^2 + 3x - 15x - 45 = 0$$

$$\boxed{x^2 - 12x - 45}$$

2. $x = 2$ and $x = \frac{4}{3}$

$$(x-2)(3x-4) = 0$$

$$3x^2 - 4x - 6x + 8$$

$$\boxed{3x^2 - 10x + 8}$$

3. $x = \frac{1}{2}$, $x = \frac{-2}{3}$, and $x = -2$

$$(2x-1)(3x+2)(x+2)$$

	$2x$	-1
$3x$	$6x^2$	$-3x$
$+2$	$4x$	-2

$$6x^2 + x - 2$$

x	$6x^3$	$+x^2$	$-2x$
$+2$	$12x^2$	$2x$	-4

$$\boxed{6x^3 + 13x^2 - 4}$$

4. $x = \pm 3i$ and $x = -5$

$$(x+3i)(x-3i)(x+5)$$

$$(x^2+9)(x+5)$$

$$x^3 + 5x^2 + 9x + 45$$

$$\boxed{x^3 + 5x^2 + 9x + 45}$$

5. $x = 1 \pm 5i$

$$(x-1+5i)(x-1-5i)$$

	x	-1	$-5i$
x	x^2	$-x$	$-5xi$
-1	$-x$	1	$5i$
$5i$	$5xi$	$-5i$	$-25i^2$

$$x^2 - 2x + 1 + 25$$

$$x^2 - 2x + 26$$

7. $x = \frac{1}{3}$ $x = -\frac{2}{5}$ $x = 4$

$$(3x-1)(5x+2)(x-4)$$

$$15x^2 + 6x - 5x - 2$$

$$15x^2 + x - 2$$

	x	x^2	$-2x$
x	$15x^3$	x^2	$-2x$
-4	$-60x^2$	$-4x$	8

$$15x^3 - 59x^2 - 6x + 8$$

6. $x = 2 \pm 3i$ and $x = 1$

$$(x-2+3i)(x-2-3i)(x-1)$$

	x	-2	$3i$
x	x^2	$-2x$	$3xi$
-2	$-2x$	4	$-6i$
$-3i$	$-3xi$	$6i$	$-9i^2$

$$x^2 - 4x + 4 + 9$$

$$(x^2 - 4x + 13)(x-1)$$

$$x^2 - 4x + 13$$

	x	x^2	$13x$
x	x^3	$-4x^2$	$13x$
-1	$-x^2$	$+4x$	-13

$$x^3 - 5x^2 + 17x - 13$$