

Multiplying and Dividing Rationals

2B

Part 1: Multiply

Steps for multiplying rational functions:

- Step #1 Factor
- Step #2 Cancellation Property
- Step #3 Multiply numerators
- Step #4 Multiply denominators
- Step #5 Product will always be in reduced form

Given $f(x) = \frac{x^2 - x - 2}{2x^2 - 3x - 2}$ and $g(x) = \frac{2x^2 - 5x - 3}{x^2 + x}$, try and find $f(x) \cdot g(x)$.

$f(x)$ factoring: $\frac{(x-2)(x+1)}{(2x+1)(x-2)}$
 $g(x)$ factoring: $\frac{(2x+1)(x-3)}{x(x+1)}$

Step #1 Factor. $f(x)$ and $g(x)$.

Step #2 Use the cancellation property.

$$\frac{(x-2)(x+1)}{(2x+1)(x-2)} \cdot \frac{(2x+1)(x-3)}{x(x+1)}$$

Step #3 What are the expressions remaining in the numerator? $x-3$

Step #4 What are the expressions remaining in the denominator? x

Step #5 Write your answer as $\frac{\text{product of numerator}}{\text{product of denominator}}$. Use any excluded values.

$$\frac{x-3}{x}$$

Practice: Multiply each expression and write your answer in reduced form.

$$1. \frac{\cancel{3x}}{5x-15} \cdot \frac{\cancel{x-3}}{\cancel{9x^2} \cdot 3x}$$

$$\boxed{\frac{1}{15x}}$$

$$2. \frac{\cancel{3ab^2}^1 \cdot \cancel{2c^2}^c}{\cancel{4c}^2 \cdot \cancel{27ab}^9} = \boxed{\frac{bc}{18}}$$

$$3. \frac{\cancel{x+5}}{x^2-4x+3} \cdot \frac{\cancel{x-3}}{4x+20}$$

$(x-3)(x-1) \quad 4(x+5)$

$$\boxed{\frac{1}{4(x-1)}}$$

$$4. \frac{\cancel{7(x-1)}}{\cancel{3x^2}} \cdot \frac{\cancel{x+5}}{9x^2-9} \cdot \frac{\cancel{(x-6)(x+1)}}{x^2-5x-6}$$

$9(x^2-1) \quad x(x^2+6x+5)$
 $9(x+1)(x-1) \quad x(x+5)(x+1)$

$$\frac{7(x-6)}{3x^2(9)(x)(x+1)}$$

or

$$\boxed{\frac{7(x-6)}{27x^3(x+1)}}$$

Part 2: Divide

$$\frac{1}{2} \div \frac{3}{4} = \frac{1}{2} \cdot \frac{4}{3}$$

Steps for dividing rational expressions:

$$\frac{1}{2} \div \frac{3}{4} = \frac{1}{2} \cdot \frac{4}{3}$$

- Step #1 Rewrite division problem as an equivalent multiplication problem
- Step #2 Factor & use the Cancellation Property
- Step #3 Multiply numerators
- Step #4 Multiply denominators
- Step #5 Product will always be in reduced form

Given $f(x) = \frac{x^2 - x - 6}{x^2 + 3x - 4}$ and $g(x) = \frac{x^2 - 5x + 6}{x^2 + 2x - 8}$, try and find $\frac{f(x)}{g(x)}$.

Step #1 Write problem as an equivalent multiplication problem.

$$\frac{x^2 - x - 6}{x^2 + 3x - 4} \cdot \frac{x^2 + 2x - 8}{x^2 - 5x + 6} \quad -6 \cdot -1$$

Step #2 Factor & use the cancellation property.

$$\frac{(\cancel{x-3})(x+2)}{(\cancel{x+4})(x-1)} \cdot \frac{(\cancel{x+4})(\cancel{x-2})}{(\cancel{x-3})(x-2)}$$

Step #3 What are the expressions remaining in the numerator?

Step #4 What are the expressions remaining in the denominator?

Step #5 Write your answer as $\frac{\text{product of numerator}}{\text{product of denominator}}$. ~~List excluded values.~~

$$\boxed{\frac{x+2}{x-1}}$$

Practice: Determine the quotient of each expression.

$$5. \frac{9ab^2}{4c} \div \frac{5ab}{18c^2} = \frac{81bc}{10}$$

*Handwritten work shows cancellation of 9, 5, and 18, leaving 3, 1, and 2 respectively, resulting in 3/2 * 18c^2 / 4c = 81bc/10.*

$$6. \frac{7x^2}{3x^2-27} \div \frac{4x^2}{3x-9} = \frac{7}{4(x+3)}$$

Handwritten work shows factoring 3x^2-27 as 3(x^2-9) = 3(x-3)(x+3) and 3x-9 as 3(x-3). The (x-3) terms cancel, leaving 7x^2 / (3(x+3)) divided by 4x^2 / 3, resulting in 7 / (4(x+3)).

$$7. \frac{3x^2+15x}{x^2-3x-40} \div \frac{5x^2}{x^2-64}$$

$$\frac{3x^2+15x}{x^2-3x-40} \cdot \frac{x^2-64}{5x^2}$$

$$\frac{3x(x+5)}{(x-8)(x+5)} \cdot \frac{(x-8)(x+8)}{5x^2}$$

$$\frac{3(x+8)}{5x}$$

$$\begin{array}{r|l} 3x-2 & \\ \hline x & 3x^2 - 2x \\ + & 21x - 14 \\ \hline & -42 \end{array}$$

Handwritten notes: 1.42, 2.21

$$8. \frac{4x}{x^2y^2-xy} \div \frac{x^2-4}{3x^2+19x-14} \div \frac{x-2}{xy}$$

$$\frac{4x}{x^2y^2-xy} \cdot \frac{(3x-2)(x+7)}{3x^2+19x-14} \cdot \frac{xy}{x-2}$$

Handwritten work shows factoring x^2y^2-xy as xy(xy-1) and 3x^2+19x-14 as (3x-2)(x+7). The (x-2) terms cancel.

$$\frac{4x(3x-2)(x+7)}{(xy-1)(x+2)(x-2)^2}$$

$$\frac{3x+3}{2x}$$

Surrender at Six: Multiply and Divide Rational Expressions

1. $\frac{x^2 + 8x + 15}{x^2} \div (x + 3)^2 =$

$$\frac{x+5}{x^2(x+3)}$$

2. $\frac{x^2 - 9}{\frac{x+4}{x-3}} =$

$$x+3$$

3. $\frac{x^2 - 16}{x - 9} \div \frac{x^2 + 14x + 40}{x^2 + x - 90} =$

$$x-4$$

$$\frac{4(2x-1)}{3}$$

$$\frac{(x+9)(x-4)}{x-9} \cdot \frac{(x+10)(x-9)}{(x+10)(x+9)}$$

$$\frac{(x-4)(x+10)}{x+10}$$

4. $(x - 5) \cdot \frac{7}{x^3 - 5x^2} =$

$$\frac{7}{x^2}$$

5. $\frac{(x+2)^2}{6x^2} \cdot \frac{3x}{x^2 - 4} =$

$$\frac{(x+2)}{2x(x-2)}$$

6. $\frac{x^2 + 7x - 8}{x + 8} \cdot \frac{x + 5}{9x - 9} =$

$$1$$

$$\frac{(x+8)(x-1)}{(x+8)} \cdot \frac{x+5}{9(x-1)}$$

$$\frac{x+5}{9}$$

Multiplying and Dividing

Homework

Name: Key

Determine the produce or quotient, list excluded values.

1. $\frac{x^2+7x+12}{12} \cdot \frac{4}{x+4}$

$$\frac{(x+3)(x+4)}{\cancel{12}_3} \cdot \frac{\cancel{4}^1}{x+4}$$

$$\boxed{\frac{x+3}{3}}$$

2. $\frac{x^2-4}{2x-4} \cdot \frac{2}{x+2}$

$$\frac{(x+2)(x-2)}{\cancel{2(x-2)}} \cdot \frac{\cancel{2}}{x+2}$$

$$\boxed{1}$$

3. $\frac{x^2+4x+3}{5x} \div \frac{x+1}{x+5}$

$$\frac{(x+3)(x+1)}{5x} \cdot \frac{x+5}{\cancel{x+1}}$$

$$\boxed{\frac{(x+3)(x+5)}{5x}}$$

4. $\frac{x^2-7x+12}{x^2-x-6} \div \frac{x^2-16}{x^2+x-2}$

$$\frac{(x-4)(x-3)}{(x-3)(x+2)} \cdot \frac{(x+2)(x-1)}{(x+4)(x-4)}$$

$$\boxed{\frac{(x-1)}{x+4}}$$

5. $\frac{x^2+6x}{6} \cdot \frac{x^3+6}{x^3+6x^2}$

$$\frac{x(x+6)}{\cancel{6}} \cdot \frac{x^3+6}{x^2(x+6)}$$

$$\frac{x(x^3+6)}{6x^2} = \boxed{\frac{x^3+6}{6x}}$$

6. $\frac{x^2-9}{x^2+x} \div \frac{x-3}{x^2-1}$

$$\frac{(x+3)(x-3)}{x(x+1)} \cdot \frac{(x+1)(x-1)}{\cancel{(x-3)}}$$

$$\boxed{\frac{(x+3)(x-1)}{x}}$$

$$7. \frac{x^2 + 2xy + y^2}{x^2 - y^2} \div \frac{x+y}{x-y}$$

$$\frac{\cancel{(x+y)}(\cancel{x+y})}{(\cancel{x+y})(x-y)} \cdot \frac{x-y}{\cancel{x+y}}$$

$$\boxed{1}$$

$$8. \frac{x^2}{x^2 - 25x^2} \cdot \frac{x^2}{-24x} \cdot x$$

$$\frac{x^2}{-24x^2} \cdot \frac{-4x}{x} = \frac{-4x}{-24x} = \boxed{\frac{1}{6}}$$

$$9. \frac{x^2 + x - 2}{x^2 - 4x - 12} \cdot \frac{x^2 - 5x - 6}{x^2 - 2x + 1}$$

$$\frac{\cancel{(x+2)}(\cancel{x-1})}{(\cancel{x-6})(x+2)} \cdot \frac{\cancel{(x-6)}(x+1)}{(\cancel{x-1})(\cancel{x+1})}$$

$$\boxed{\frac{x+1}{x-1}}$$

$$10. \frac{(x+3)^2}{x^2 - 16} \cdot \frac{x+4}{x+3}$$

$$\frac{\cancel{(x+3)}(x+3)}{\cancel{(x+4)}(x-4)} \cdot \frac{\cancel{x+4}}{x+3}$$

$$\boxed{\frac{x+3}{x-4}}$$

$$11. \frac{x^2 + 3x - 10}{x^2 - 7x + 6} \cdot \frac{x^2 + 2x - 3}{x^2 + x - 6}$$

$$\frac{\cancel{(x+5)}(\cancel{x-2})}{(\cancel{x-6})(\cancel{x-1})} \cdot \frac{\cancel{(x+3)}(\cancel{x-1})}{\cancel{(x+3)}(\cancel{x-2})}$$

$$\boxed{\frac{x+5}{x-6}}$$