EXPLORE
Algebra II
Linear Programming

Name: key
Period: _____

Today is the last day of your vacation in Mexico. You have enough space in your carry-on bag for no more than 20 gifts for your family and friends. Mini sombreros cost $3 and maracas cost $6. You have a budget of $90 cash.

Write a system of inequalities that represent the situation. Graph and shade the grid appropriately.

x = # of Sombreros
y = # Maracas

$x + y \leq 20$

Gifts

$y \leq -x + 20$

pink

$

$3x + 6y \leq 90$

$-3x \quad -3x$

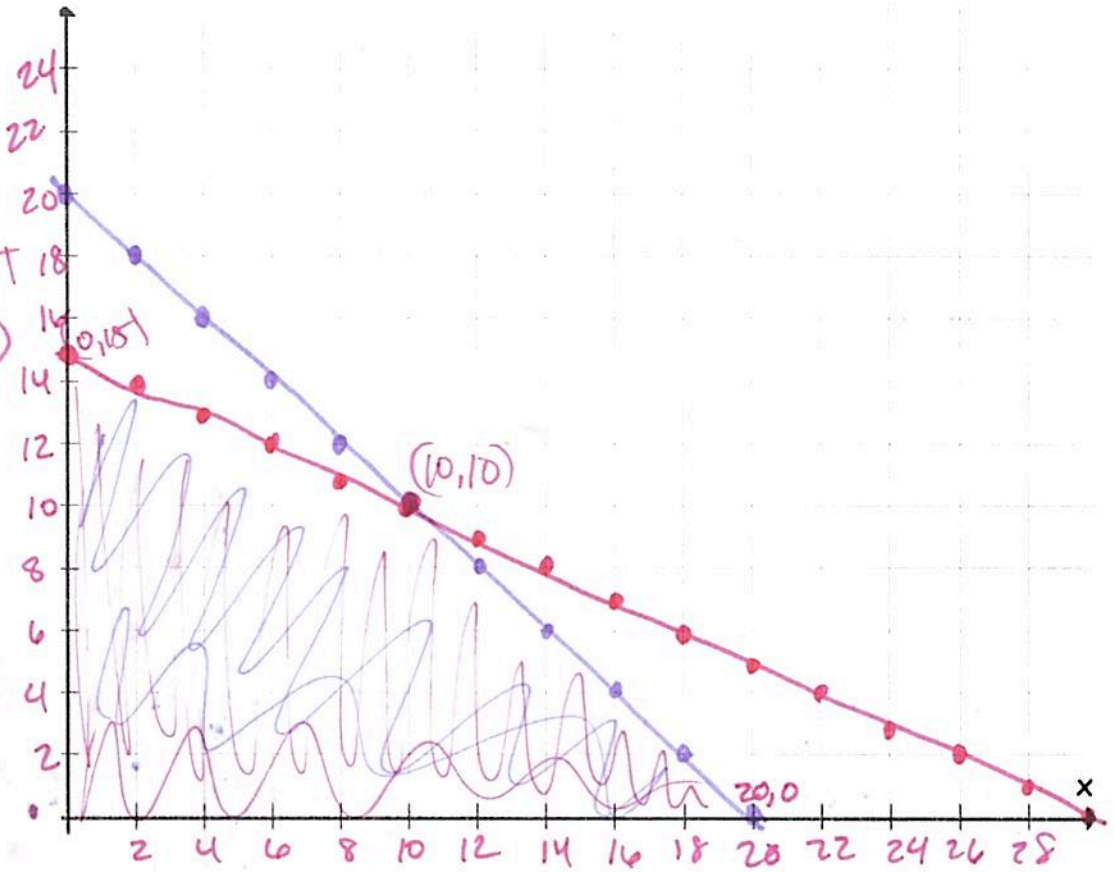
$\frac{6y}{6} \leq \frac{-3x + 90}{6}$

$y \leq -\frac{1}{2}x + 15$

purple

$x \geq 0$

$y \geq 0$



List 3 combinations of mini sombreros and maracas you could buy.

(20,0)
S, M

(0,15)
S, M

(10,10)
S, M

* Parameters *

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Shipping it Out

The Supreme Shipping Company can load its trucks with both rectangular and cylindrical containers. A rectangular container has a volume of 100 cubic ft and weighs 200 lb. A cylindrical container has a volume of 200 cubic feet and weighs 100 lb. Let x denote the number of rectangular crates carried by a truck, and let y denote the number of cylindrical containers.

1. What constraint must be satisfied if each truck has room for at most 4200 cubic ft of containers?

$100x + 200y \leq 4200$

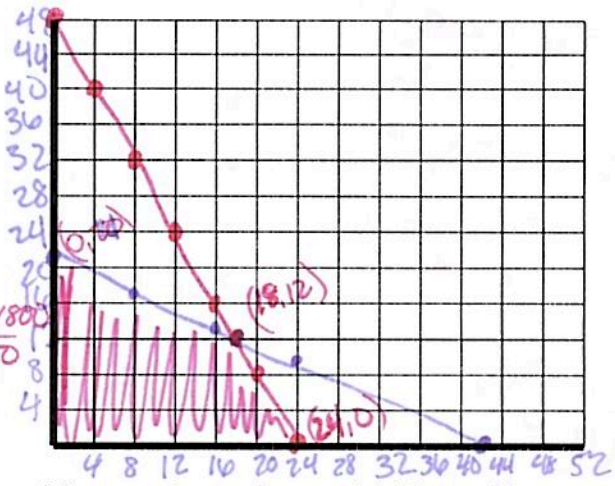
2. What constraint must be satisfied if each truck can carry a maximum of 4800 lbs?

$200x + 100y \leq 4800$

3. What additional constraints must be satisfied because the problem involves real objects?

$x \geq 0, y \geq 0$

4. Graph the feasibility set on the grid and label its vertices. Call the vertex on the x-axis A, the vertex on the y-axis B, and the vertex on neither axis C.



$100x + 200y \leq 4200$
$-100x \quad -100x$
$\frac{200y}{200} \leq \frac{-100x}{200} + \frac{4200}{200}$
$y \leq -\frac{1}{2}x + 21$

$200x + 100y \leq 4800$
$-200x \quad -200x$
$\frac{100y}{100} \leq \frac{-200x}{100} + \frac{4800}{100}$
$y \leq -2x + 48$

5. Suppose that Supreme Shipping charges $50 to ship a rectangular and $60 to ship a cylindrical container and wishes to maximize its income.

   a. What is the objective function? $P = 50x + 60y$

   b. What is the value of the objective function at vertex A? (0,20) ↳ x-axis $50(0) + 60(21) = 1260$

   c. At vertex B? (y-axis) $50(18) + 60(12) = 1620$

   d. At vertex C? (neither) $50(24) + 60(0)$

   e. What combination of containers should Supreme Shipping use to maximize its income?

18 Rectangular + 12 Cylindrical

ELABORATE
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Problem Solving: Linear Programming

1. Each quart of the regular skin lotion that Harold makes contains 2 cups of oil and 1 cup of cocoa butter. Each quart of his extra rich lotion contains 1 cup of oil and 2 cups of cocoa butter. Harold makes $10 profit on each quart of the regular lotion and an $8 profit on each quart of the extra rich lotion. If he has 12 cups of oil and 8 cups of cocoa butter on hand, how many quarts of each type of lotion should he make to maximize his profits?

Define Variables:

X = Regular

y = extra Rich

Objective Function:

$P = 10x + 8y$

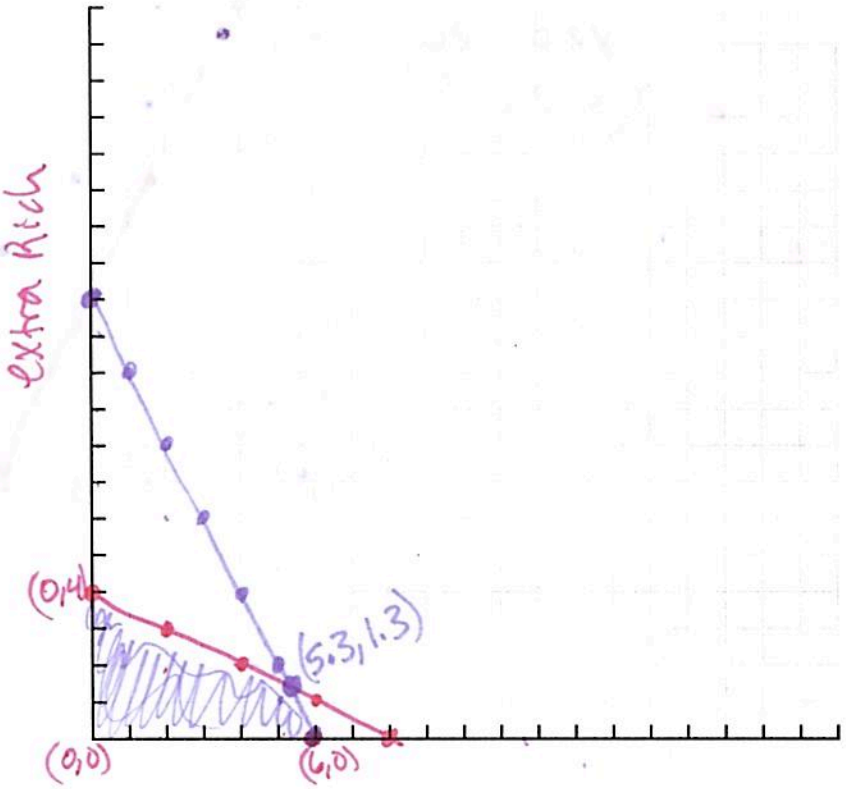
Constraints:

$x \geq 0$

$y \geq 0$

oil $2x + y \leq 12 \rightarrow y \leq -2x + 12$

co $x + 2y \leq 8 \rightarrow y = -\frac{1}{2}x + 4$



Vertices:

(0,0) (0,4)
(6,0)
(5.3, 1.3)

Substitute into the objective function:

$10(0) + 8(0) = 0$

$10(6) + 8(0) = 60$

$10(5.3) + 8(1.3) = 63.4$

$10(0) + 8(4) = 32$

Solution in words:

You will need 5.3 quarts of regular lotion and 1.3 quarts of extra Rich lotion.

2. A loaf of Irish soda bread requires 4 cups of flour and 1 cup of sugar. A loaf of zucchini bread uses 2 cups of flour and 1 cup sugar. Mari Alice has 16 cups of flour and 7 cups of sugar on hand. She makes $2 profit per loaf of Irish soda bread and $3 per loaf of zucchini bread. To maximize profits, how many loaves of each type should she make?

Define Variables:

x = ISB loaves

y = ZB loaves

Objective Function:

$P = 2x + 3y$

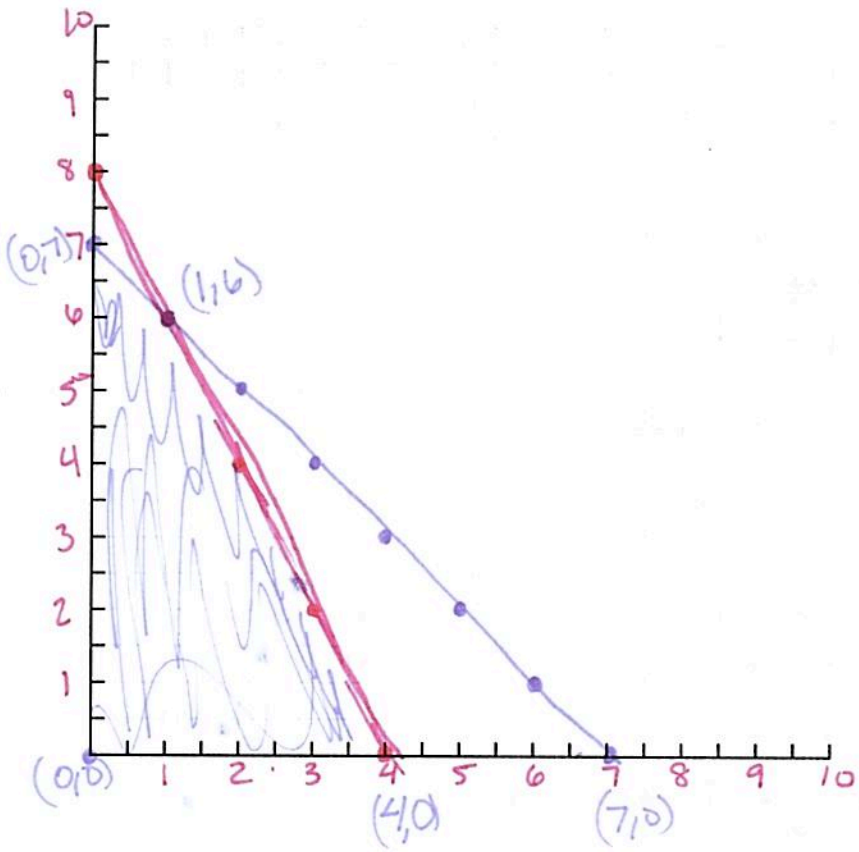
Constraints: $x \geq 0$ $y \geq 0$

flour $4x + 2y \leq 16$ $y \leq -2x + 8$

sugar $1x + y \leq 7$ $y \leq -x + 7$

Vertices:

(0,0) (0,7) (1,6) (4,0)



Substitute into the objective function:

$2(0) + 3(0) = 0$

$2(0) + 3(7) = 21$ ✓

$2(1) + 3(6) = 20$

$2(4) + 3(0) = 8$

Solution in words:

Profits would be maximized by making 7 loaves of Zucchini Bread and No Irish Soda Bread.

Linear Programming Day Name: Key

Evaluate

Use linear programming. Find the values of x and y that maximize or minimize each objective function.

1. It takes a tailoring shop 2 hours of cutting and 4 hours of sewing to make a knit suit. To make a wool suit, it takes 4 hours of cutting and 2 hours of sewing. At most, 200 hours each day are available for cutting, and 160 hours are available for sewing. The shop makes a profit of $34 on each knit suit and $31 on each wool suit. How many of each type suit should they make in order to maximize the profit?

Variables: x = knit
y = wool

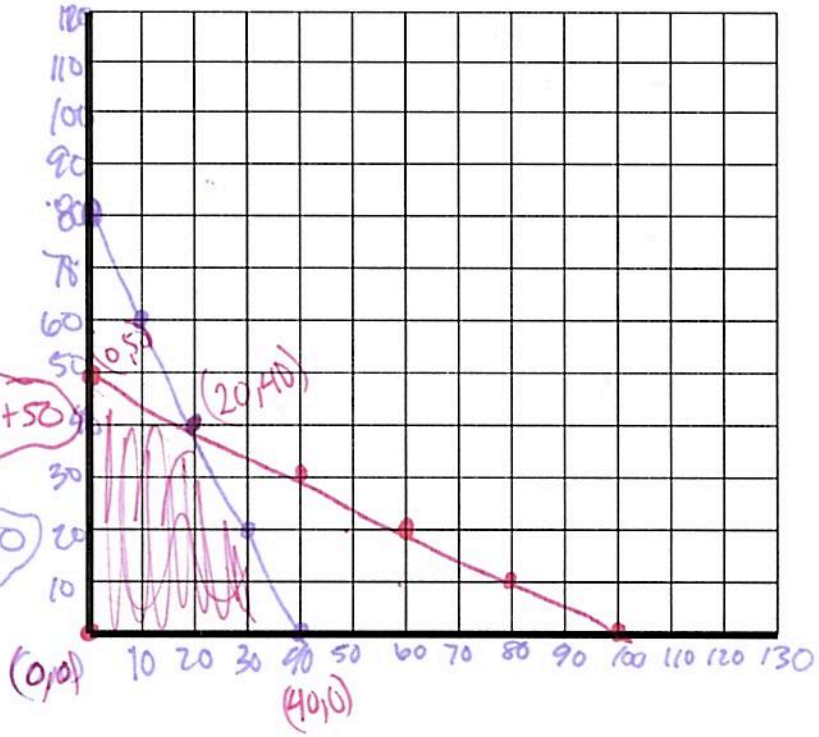
Objective Function:

$P = 34x + 31y$

Constraints: $x \geq 0, y \geq 0$

$2x + 4y \leq 200 \rightarrow y \leq -\frac{1}{2}x + 50$

$4x + 2y \leq 160 \rightarrow y \leq -2x + 80$



Vertices:

(0,0), (0,50), (20,40), (40,0)

Substitute into the objective function:

$34(0) + 31(0) = 0$
$34(0) + 31(50) = 1550$
$34(20) + 31(40) = 1920$ ✓
$34(40) + 31(0) = 1360$

How many of each suit should be made to maximize the profit?

20 knit, 40 wool

2. A farmer has no more than 50 acres for planting alfalfa and soybeans and has a maximum of $1200 to spend for planting. It cost $20 per acre to plant alfalfa and $30 per acre to plant soybeans. The profit per acre for alfalfa is $250 and the profit for soybeans is $300. How many acres of each crop should he plant in order to maximize his profit?

x y

Variables:

x = Alfalfa

y = Soybeans

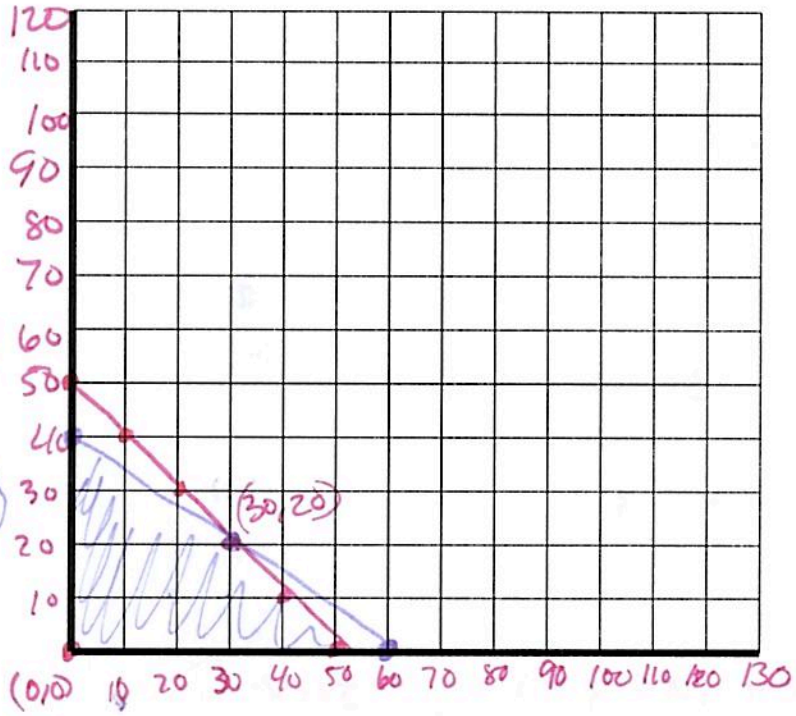
Objective Function:

$P = 250x + 300y$

Constraints: $x \geq 0, y \geq 0$

$x + y \leq 50 \rightarrow y \leq -x + 50$

$20x + 30y \leq 1200 \rightarrow y \leq -\frac{2}{3}x + 40$



Vertices:

(0,0), (0,40), (30,20), (50,0)

Substitute into the objective function:

$250(0) + 300(0) = 0$

$250(0) + 300(40) = 12{,}000$

$250(30) + 300(20) = 13{,}500$ ✓

$250(50) + 300(0) = 12{,}500$

How many acres of each crop should be planted to maximize profit?

30 acres of Alfalfa + 20 acres of Soybeans