

12.3/12.4 Solving Exponential & Log Equations

Solve the following. Do not use a calculator (leave your answers exact).

1.  $\left(\frac{1}{8}\right)^x = 64$   
 $(2^{-3})^x = 2^6$   
 $-3x = 6$   
 $\frac{-3x}{-3} = \frac{6}{-3}$   
 $x = -2$

2.  $6(10^x) = 216$   
 $\frac{6(10^x)}{6} = \frac{216}{6}$   
 $10^x = 36$

$\log_{10}(36) = x$

3.  $2^{x+3} = 256$   
 $2^{x+3} = 2^8$   
 $x+3 = 8$   
 $x = 5$

4.  $3^{x-1} = \frac{1}{81}$   
 $3^{x-1} = 3^{-4}$   
 $x-1 = -4$   
 $x = -3$

5.  $\ln x - \ln 5 = 0$   
 $\ln\left(\frac{x}{5}\right) = 0$   
 $e^0 = \frac{x}{5}$   
 $1 = \frac{x}{5}$   
 $5 = x$

6.  $\ln(2x-1) = 5$   
 $e^5 = 2x-1$   
 $\frac{e^5 + 1}{2} = x$

7.  $\log_3 x + \log_3 4 = \frac{1}{4} \log_3 81$   
 $\log_3(4x) = \log_3 \sqrt[4]{81}$   
 $4x = 3$   
 $x = \frac{3}{4}$

8.  $-1 + 2 \ln 3x = 17$   
 $+1 \quad +1$   
 $2 \ln 3x = 18$   
 $\ln 3x = 9$   
 $e^9 = \frac{3x}{3}$   
 $x = \frac{e^9}{3}$   
 $4^{1/2} = \frac{x}{x-1} \Rightarrow 2 = \frac{x}{x-1}$   
 $x = 2x - 2$   
 $x = 2$

9.  $\log_4 x - \log_4(x-1) = \frac{1}{2}$   
 $\log_4 \frac{x}{x-1} = \frac{1}{2}$

Solve the following. Use calculator to check your answer. Truncate your result to three decimal places.

10.  $500e^{-x} = 300$   
 $\frac{500e^{-x}}{500} = \frac{300}{500}$   
 $e^{-x} = \frac{3}{5}$   
 $\ln\left(\frac{3}{5}\right) = -x$   
 $\frac{\ln(3/5)}{-1} = \frac{-x}{-1}$   
 $x = .5108$

11.  $7 - 2e^x = 5$   
 $-7 \quad -7$   
 $-2e^x = -2$   
 $\frac{-2e^x}{-2} = \frac{-2}{-2}$   
 $e^x = 1$   
 $\ln(1) = x$   
 $x = 0$

$$12. 5(2^{3-x}) - 13 = 100$$

$$\frac{5(2^{3-x})}{5} = \frac{113}{5}$$

$$2^{3-x} = 22.6$$

$$\log_2(22.6) = 3 - x$$

$$1.498 = -x$$

$$\boxed{-1.498 = x}$$

$$13. \log_6(3x) + \log_6(x-1) = 3$$

$$\log_6(3x(x-1)) = 3$$

$$\log_6(3x^2 - 3x) = 3$$

$$6^3 = 3x^2 - 3x$$

$$216 = 3x^2 - 3x - 216$$

$$0 = 3(x^2 - x - 72)$$

$$\boxed{14}$$

$$0 = 3(x-9)(x+8)$$

$$\boxed{x=9}$$

$x = -8$   
extr

$$14. \frac{\ln(x+19)}{\ln(7x-8)} = 1$$

$$\ln(x+19) = \ln(7x-8)$$

$$x+19 = 7x-8$$

$$19 = 6x - 8$$

$$\frac{27}{6} = \frac{6x}{6}$$

$$\boxed{x=4.5}$$

$$15. \ln 4x = 2.1$$

$$\frac{e^{2.1}}{4} = \frac{4x}{4}$$

$$\boxed{x = 2.041}$$

$$\boxed{x = \frac{e^{2.1}}{4}}$$

$$16. \ln(x+5) = \ln(x-1) - \ln(x+1)$$

$$\ln(x+5) = \ln\left(\frac{x-1}{x+1}\right)$$

$$\frac{x+5}{1} = \frac{x-1}{x+1}$$

$$0 = x^2 + 5x + 6$$

both  
(x+3)(x+2) extraneous

$x = -2$   $x = -3$

$$x-1 = (x+5)(x+1)$$

$$x-1 = x^2 + 5x + x + 5$$

$$x-1 = x^2 + 6x + 5$$