

Engage: Factoring and Graphing Polynomials

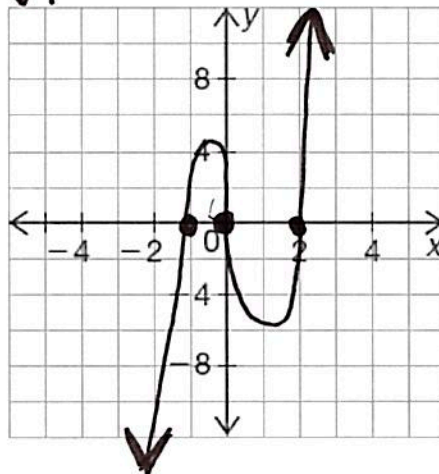
Factor each expression. Remember to look for a greatest common factor first. Then, use the factors to sketch the graph of each polynomial.

a.  $3x^3 - 3x^2 - 6x$

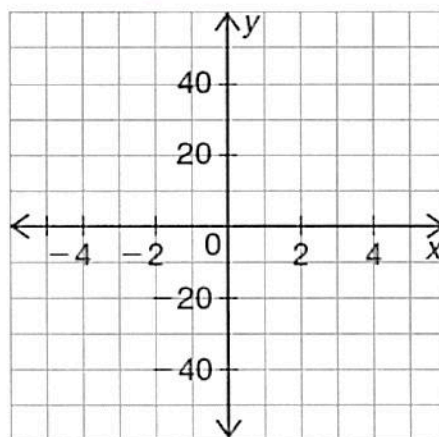
$3x(x^2 - x - 2)$

$3x(x - 2)(x + 1)$

$3x = 0 \rightarrow x = 0$   
 $x - 2 = 0 \rightarrow x = 2$   
 $x + 1 = 0 \rightarrow x = -1$



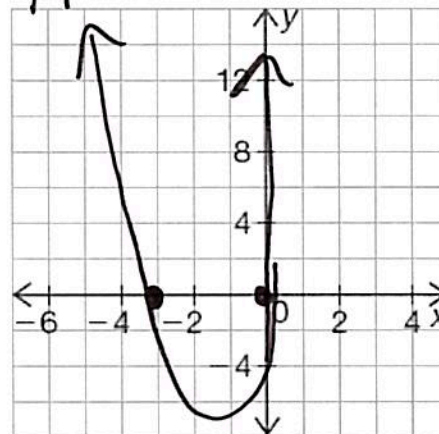
b.  $x^3 - x^2 - 20x$



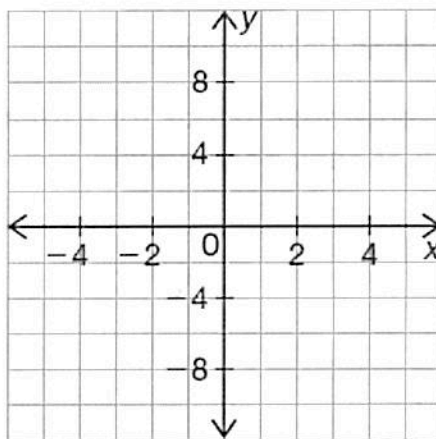
c.  $2x^2 + 6x$

$2x(x + 3)$

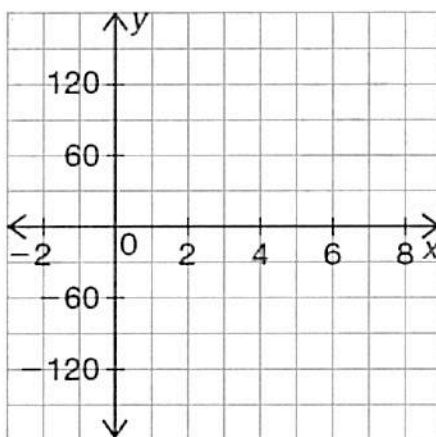
$2x = 0 \rightarrow x = 0$   
 $x + 3 = 0 \rightarrow x = -3$



d.  $3x^2 - 3x - 6$



e.  $10x^2 - 50x - 60$



## 5.4 Factoring Higher Order Polynomials

Factor completely

### 1. Factoring Difference of Squares

$$x^2 - 64 \quad a^2 = x^2 \quad b^2 = 64$$

$$a = x \quad b = 8$$

$$(x+8)(x-8)$$

$$a^2 - b^2 = (a+b)(a-b)$$

$$25x^2 - 16$$

$$(5x+4)(5x-4)$$

$$a = 5x \quad b = 4$$

$$x^8 - 1 \quad a = x^4 \quad b = 1$$

$$(x^4+1)(x^4-1) \quad a = x^2 \quad b = 1$$

$$(x^4+1)(x^2+1)(x^2-1)$$

$$(x^4+1)(x^2+1)(x+1)(x-1)$$

$$x^4 - y^4$$

$$(x^2+y^2)(x^2-y^2)$$

$$(x^2+y^2)(x+y)(x-y)$$

### 2. Factoring Trinomials (Can use Box method)

$$x^2 - 10x + 9$$

$$(x-9)(x-1)$$

$$x^4 - 10x^2 + 9$$

$$(x^2-9)(x^2-1)$$

$$(x+3)(x-3)(x+1)(x-1)$$

$x^2 - 1$	
$x^2$	$x^2$
$x^4$	$-10x^2$
$-9x^2$	$9$

$$x^2 - 29x + 100$$

$$(x-25)(x-4)$$

$$x^4 - 29x^2 + 100$$

$$(x^2-25)(x^2-4)$$

$$(x+5)(x-5)(x+2)(x-2)$$

$x^2 - 4$	
$x^2$	$x^2$
$x^4$	$-4x^2$
$-25x^2$	$100$

$$16x^2 - 80x + 100$$

$$4(4x^2 - 20x + 25)$$

$$4(2x-5)(2x-5)$$

$$4(2x-5)^2$$

$2x - 5$	
$2x$	$5$
$4x^2$	$-10x$
$-10x$	$25$

$$9x^2 + 6x + 1$$

$$(3x+1)^2$$

$3x + 1$	
$3x$	$1$
$9x^2$	$6x$
$6x$	$1$

$$6x^4 - 15x^3 + 6x^2$$

$$3x^2(2x^2 - 5x + 2)$$

$$3x^2(2x-1)(x-2)$$

### 3. Factoring by Grouping, continue to check for GCF.

Can use  
Box  
after GCF

$$(10x^2 - 15x) + (8x - 12)$$

Grouping

$$5x(2x-3) + 4(2x-3)$$

$$(5x+4)(2x-3)$$

$$(2x^3 + 6x^2) - (2x - 6)$$

Grouping

~~2x^2(x+3)~~

$$2(x^3 + 3x^2 - x - 3)$$

$$2[x^2(x+3) - 1(x+3)]$$

$$2(x^2 - 1)(x+3)$$

$$2(x+1)(x-1)(x+3)$$

$$20x^4 + 12x^3 + 10x^2 + 6x$$

Grouping

$$2x[10x^3 + 6x^2 + 5x + 3]$$

$$2x[(2x^2(5x+3) + 1(5x+3))]$$

$$(2x)(2x^2+1)(5x+3)$$

$$x^4 - 4x^3 - x^2 + 4x$$

Grouping

$$x[x^3 - 4x^2 - x + 4]$$

$$x[x^2(x-4) - (x-4)]$$

$$(x)(x^2-1)(x-4)$$

$$(x)(x+1)(x-1)(x-4)$$

#### 4. Factoring the sum and difference of cubes.

\*There is a formula, see page 451 in your book to see why these formulas work. This formula is on your formula chart.

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$\downarrow$  same      $\downarrow$  opp.      $\downarrow$  way to pos.

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

same      $\downarrow$  1     all pos

Remember your signs with **SOAP**: Same Opposite Always Positive

$$8x^3 - 125 \quad a=2x \quad b=5$$

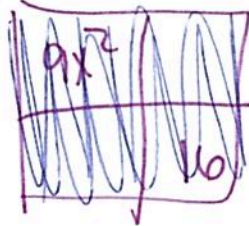
$$(2x-5)(4x^2 + 10x + 25)$$

$$x^3 + 27 \quad a=x \quad b=3$$

$$(x+3)(x^2 - 3x + 9)$$

$$27x^3 - 64 \quad a=3x \quad b=4$$

$$(3x-4)(9x^2 + 12x + 16)$$



$$125x^3 + 1 \quad a=5x \quad b=1$$

$$(5x+1)(25x^2 - 5x + 1)$$

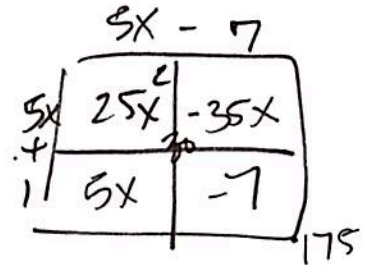
Practice Problems: Factor completely!

1.  $x^3 + 64 \quad a=x \quad b=4$

$$(x+4)(x^2 - 4x + 16)$$

2.  $25x^2 - 30x - 7$

$$(5x-7)(5x+1)$$



3.  $27x^3 - 64 \quad a=3x \quad b=4$

$$(3x-4)(9x^2 + 12x + 16)$$

4.  $24x^4 - 8x^3 + 21x^2 - 7x$

$$x(8x^2 + 7)(3x-1)$$



## Evaluate: Factoring Higher Order Polynomials

Polynomial	Method of Factoring	Reason	Factored Form
$3x^4 + 2x^2 - 8$	"Trinomial" Box	3 terms	$(x^2+2)(3x^2-4)$
$9x^2 - 16$	Difference of Squares	$(3x+4)(3x-4)$	
$x^2 - 12x + 36$	"Trinomial" Box	$(x-6)^2$	
$x^3 - 64$ $a=x$ $b=4$	Difference of Cubes	$(x-4)(x^2+4x+16)$	
$x^3 + 2x^2 + 7x + 14$	Grouping	4 terms $x^2(x+2)+7(x+2)$	$(x^2+7)(x+2)$
$25x^2 - 30x - 7$	Box	$(5x+1)(5x-7)$	
$2x^4 + 10x^3 + 12x^2$	Box	$2x^2(x+3)(x+2)$ $2x^2(x^2+5x+6)$	





1.  $3x^4 + 2x^2 - 8$

$$(3x^2 - 4)(x^2 + 2)$$

		$3x^2 - 4$	
$x^2$	$3x^4$	$-4x^2$	$x^2$
+	$2x^2$	$-8$	
2	$6x^2$	$-8$	$-24$

2.  $9x^2 - 16$      $a = 3x$      $b = 4$

$$(3x + 4)(3x - 4)$$

3.  $x^2 - 12x + 36$

$$(x - 6)(x - 6)$$

$$(x - 6)^2$$

4.  $x^3 + 2x^2 + 7x + 14$

$$x^2(x + 2) + 7(x + 2)$$

$$(x^2 + 7)(x + 2)$$

5.  $2x^4 + 10x^3 + 12x^2$

$$2x^2(x^2 + 5x + 6)$$

$$2x^2(x + 3)(x + 2)$$

$$2x^2(x + 3)(x + 2)$$

6.  $12x^5 + 48x^4 + 10x^3 + 40x^2$

$$2x^2(6x^3 + 24x^2 + 5x + 20)$$

$$2x^2[6x^2(x + 4) + 5(x + 4)]$$

$$2x^2(6x^2 + 5)(x + 4)$$

