

Factor Theorem

Let a polynomial $f(x)$ be divided by $x - a$. If the remainder is 0, then $x - a$ is a factor of $f(x)$.

Ex 1. Let $f(x) = 3x^3 - 4x^2 - 28x - 16$. Is $(x+2)$ a factor?

Ex 2: Synthetic division can also be used to help us factor and solve polynomials.

Given that $(x + 3)$ is a factor of $f(x) = 2x^3 + 9x^2 + 10x + 3$.

Step 1: Divide out the factor.

Step 2: Factor the Quadratic expression left.

Step 3: List all three factors.

Step 4: Determine the zeros, or roots.

Ex 3: Factor $f(x) = x^3 - 18x^2 + 95x - 126$
given that $(x - 9)$ is a factor.

Ex 4: Factor $f(x) = 2x^3 + 3x^2 - 39x - 20$
given that -5 is a zero of $f(x)$.

Ex 5: Show that $(3x-2)$ is a factor of $3x^3-8x^2+16x-8$. Can you use Synthetic?

Remainder Theorem

If a polynomial $f(x)$ is divided by $x - a$, the remainder is the constant $f(a)$.

So to find $f(a)$ you can do synthetic division of the polynomial with the "a" value.

And,

$f(x) = q(x) \cdot (x - a) + f(a)$ where $q(x)$ is a polynomial with degree one less than the degree of $f(x)$.

$$\begin{array}{r} q(x) \\ x-a \overline{) f(x)} \\ \hline \text{WORK} \\ f(a) \end{array}$$

To find the y-value for $f(a)$, divide $f(x)$ by $(x-a)$ and your remainder is the answer.

Ex 6: Let $f(x) = 2x^4 - x^3 + 4$. Show that $f(-1)$ is the remainder when $f(x)$ is divided by $(x + 1)$.

Ex 7: Let $f(x) = x^3 + 5x^2 - 7x + 2$. Find $f(2)$.