

### Practice Problems

Simplify the following expressions.

1.  $3\sqrt{6} + \sqrt{24}$

$$\begin{array}{c} \sqrt{6} \cdot \sqrt{4} \\ 3\sqrt{6} + 2\sqrt{6} = \boxed{5\sqrt{6}} \end{array}$$

2.  $10\sqrt[3]{x} - 4\sqrt[3]{x}$

$$\boxed{6\sqrt[3]{x}}$$

3.  $\frac{4}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}} = \boxed{\frac{4\sqrt{13}}{13}}$

4.  $4\sqrt{5} + \sqrt{125} + \sqrt{45}$

$$\begin{array}{c} 4\sqrt{5} + \sqrt{25} \cdot \sqrt{5} + \sqrt{9} \cdot \sqrt{5} \\ 4\sqrt{5} + 5\sqrt{5} + 3\sqrt{5} = \boxed{12\sqrt{5}} \end{array}$$

5.  $3\sqrt[3]{xy} \cdot 7\sqrt[3]{y^2}$

$$3x^{1/3}y^{1/3} \cdot 7y^{2/3}$$

$$21\sqrt[3]{xy^3}$$

$$21x^{1/3}y^{3/3}$$

$$\boxed{21y\sqrt[3]{x} \text{ or}}$$

$$\boxed{21yx^{1/3}}$$

6.  $\sqrt{3}(5\sqrt{3} - 2\sqrt{6})$

$$5\sqrt{9} - 2\sqrt{18}$$

$$5 \cdot 3 - 2\sqrt{9} \cdot \sqrt{2}$$

$$15 - 2 \cdot 3\sqrt{2} = \boxed{15 - 6\sqrt{2}}$$

7.  $\frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$$\frac{2\sqrt{2}}{2} = \boxed{\sqrt{2}}$$

8.  $2\sqrt{x} \cdot 5\sqrt{x} = 10\sqrt{x^2} = \boxed{10x}$

9.  $4\sqrt{y} + 6\sqrt{x} + 8\sqrt{x} - 7\sqrt{y}$

$$\boxed{-3\sqrt{y} + 14\sqrt{x}}$$

10.  $2\sqrt{y} \cdot 3\sqrt{y} \cdot \sqrt{y}$

$$6\sqrt{y^2} = 6y \cdot \sqrt{y} = \boxed{6y\sqrt{y}}$$

11.  $\frac{6}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = \frac{6\sqrt{10}}{10} = \boxed{\frac{3\sqrt{10}}{5}}$

12.  $8\sqrt{x} + 4\sqrt{x} - 2\sqrt{x}$

$$\boxed{10\sqrt{x}}$$

13.  $\frac{5}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \boxed{\frac{5\sqrt{7}}{7}}$

14.  $3\sqrt[3]{6} + \sqrt[3]{48}$

$$3\sqrt[3]{6} + \sqrt[3]{8} \sqrt[3]{6}$$

$$3\sqrt[3]{6} + 2\sqrt[3]{6}$$

$$\boxed{5\sqrt[3]{6}}$$

Wj

PAP Algebra 2  
10.5 Radical Operations

Name: \_\_\_\_\_

You can use the distributive property to simplify sums and differences of radical expressions when the expressions have the same radicand.

Sum:  $\sqrt{2} + 3\sqrt{2} = (1 + 3)\sqrt{2} = 4\sqrt{2}$

Difference:  $\sqrt{2} - 3\sqrt{2} = (1 - 3)\sqrt{2} = -2\sqrt{2}$

**EXAMPLE 1** *Adding and Subtracting Radicals*

a.  $2\sqrt{2} + \sqrt{5} - 6\sqrt{2} = -4\sqrt{2} + \sqrt{5}$  Subtract like radicals.

b.  $4\sqrt{3} - \sqrt{27} = 4\sqrt{3} - \sqrt{9 \cdot 3}$  Perfect square factor

$= 4\sqrt{3} - \sqrt{9} \cdot \sqrt{3}$  Use product property.

$= 4\sqrt{3} - 3\sqrt{3}$  Simplify.

$= \sqrt{3}$  Subtract like radicals.

**EXAMPLE 2** *Multiplying Radicals*

a.  $\sqrt{2} \cdot \sqrt{8} = \sqrt{16}$  Use product property.

$= 4$  Simplify.

b.  $\sqrt{2}(5 - \sqrt{3}) = 5\sqrt{2} - \sqrt{2} \cdot \sqrt{3}$  Use distributive property.

$= 5\sqrt{2} - \sqrt{6}$  Use product property.

**EXAMPLE 3** *Dividing Radicals (Rationalizing the Denominator)*

a.  $\frac{3}{\sqrt{5}} = \frac{3}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$  Multiply numerator and denominator by  $\sqrt{5}$ .

$= \frac{3\sqrt{5}}{\sqrt{5} \cdot \sqrt{5}}$  Multiply fractions.

$= \frac{3\sqrt{5}}{5}$  Simplify.

# Equation form of an Exponential Function:

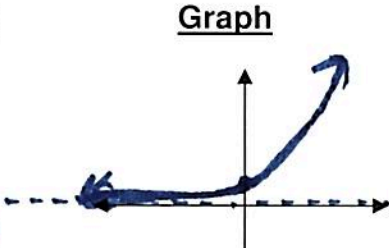
$$y = a(b)^x \quad \text{OR} \quad y = a \cdot b^x$$

$a =$  initial       $b =$  Rate  $\Rightarrow$  Growth or Decay       $x =$  time or #times

What is the difference between exponential growth and exponential decay?

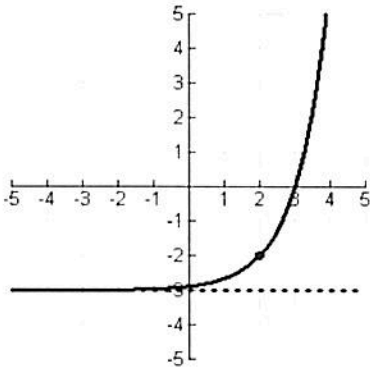
## Exponential Growth

Domain:  $\mathbb{R}$   
 Range:  $y > 0$   
 H. Asymptote:  $y = 0$   
 Critical Point:  $(0, 1)$



$$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$$

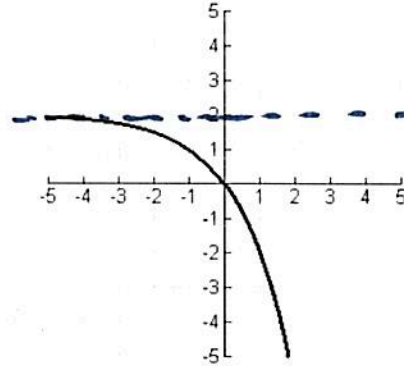
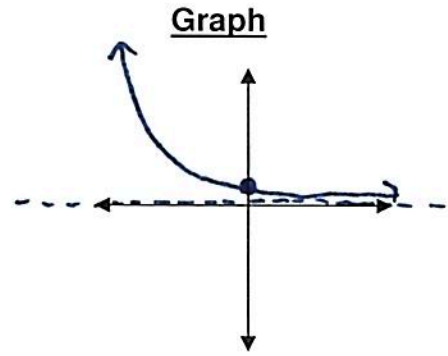
$$2^{-1000} = \frac{1}{2^{1000}} =$$



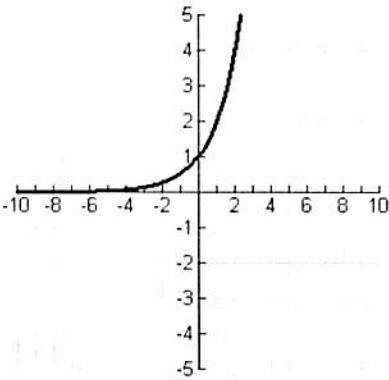
Domain:  $\mathbb{R}$   
 Range:  $y > -3$   
 Asymptote:  $y = -3$

## Exponential Decay

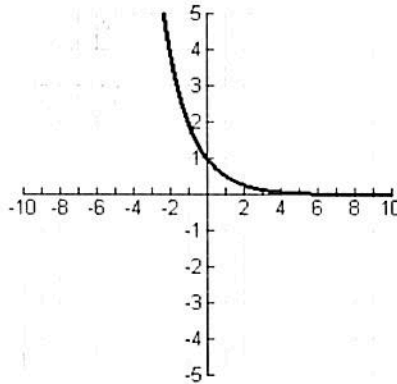
Domain:  $\mathbb{R}$   
 Range:  $y > 0$   
 H. Asymptote:  $y = 0$   
 Critical Point:  $(0, 1)$



Domain:  $\mathbb{R}$   
 Range:  $y < 2$   
 Asymptote:  $y = 2$



Domain:  $\mathbb{R}$   
 Range:  $y > 0$   
 Asymptote:  $y = 0$



Domain:  $\mathbb{R}$   
 Range:  $y > 0$   
 Asymptote:  $y = 0$

## Summary

U. Stretch  
V. Compress.  
V. Reflection

Initial  $\rightarrow$   $y = a(b)^{x+h} + k$   $\leftarrow$  Growth/Decay  
 $\leftarrow$  Up (+k)  
 $\leftarrow$  Down (-k)

$b > 1$  Growth      Left (+h)  
 $b < 1$  Decay      Right (-h)

**\*\* b is NOT A transformation \*\***



# Transformations of Exponentials

## Explain

Explain the transformations which will take place in each of the situations, state the domain and range AND show how each equation would be input into the calculator.

<p>1. <math>y = (2)^x</math></p> <p>Growth/Decay:</p> <p>Transformations:</p> <p>Critical Point:</p> <p>Asymptote:</p> <p>Domain:</p> <p>Range:</p>	<p>2. <math>y = 2^x + 2</math></p> <p>Growth/Decay:</p> <p>Transformations:</p> <p>Critical Point:</p> <p>Asymptote:</p> <p>Domain:</p> <p>Range:</p>	<p>5. <math>y = 2^{x-1} - 2</math>    <math>y = 1(2)^{x-1} - 2</math></p> <p><u>Growth</u> Decay:</p> <p>Transformations: Right + 1 Down 2</p> <p>Critical Point: (1, -1)    (0, 1)</p> <p>Asymptote: <math>y = -2</math>    <math>\frac{+1 - 2}{1, -1}</math></p> <p>Domain: <math>\mathbb{R}</math></p> <p>Range: <math>y &gt; -2</math></p>
<p>6. <math>y = 3(2)^x</math></p> <p>Growth/Decay:</p> <p>Transformations:</p> <p>Critical Point:</p> <p>Asymptote:</p> <p>Domain:</p> <p>Range:</p>	<p>9. <math>y = 4\left(\frac{1}{2}\right)^{x+3} + 0</math></p> <p>Growth/<u>Decay</u>:</p> <p>Transformations: V. stretch left + 3</p> <p>Critical Point: (-3, 4)    (0, 1)</p> <p>Asymptote: <math>y = 0</math>    <math>\frac{x \ 4}{0, 4}</math></p> <p>Domain: <math>\mathbb{R}</math>    <math>\frac{-3}{(-3, 4)}</math></p> <p>Range: <math>y &gt; 0</math></p>	<p>11. <math>y = -3(2)^x</math></p> <p>Growth/Decay:</p> <p>Transformations:</p> <p>Critical Point:</p> <p>Asymptote:</p> <p>Domain:</p> <p>Range:</p>
<p>13. <math>y = 3(2)^{x-1}</math></p> <p>Transformations: left 3 down 4</p> <p>New function:</p> <p>Critical Point:</p> <p>Asymptote:</p> <p>Domain:</p> <p>Range:</p>	<p>14. <math>y = 4(2)^{x+3} + 2</math></p> <p>Transformations: reflect over x-axis up 5</p> <p>New function: <math>y = -4(2)^{x+3} + 2</math> <span style="border: 1px solid black; padding: 2px;"><math>y = -4(2)^{x+3} + 3</math></span></p> <p>Critical Point: (-3, -1)    (0, 1)</p> <p>Asymptote: <math>y = 3</math>    <math>\frac{x \ -4}{0, -4}</math></p> <p>Domain: <math>\mathbb{R}</math>    <math>\frac{-3 \ +3}{-3, -1}</math></p> <p>Range: <math>y &lt; 3</math></p>	<p>15. <math>y = 5(2)^{x-1} - 2</math></p> <p>Transformations: right 4 up 2</p> <p>New function: <math>y = 5(2)^{x-5}</math></p> <p>Critical Point: (5, 5)    (0, 1)</p> <p>Asymptote: <math>y = 0</math>    <math>\frac{x \ 5}{0, 5}</math></p> <p>Domain: <math>\mathbb{R}</math>    <math>\frac{+5}{5, 5}</math></p> <p>Range: <math>y &gt; 0</math></p>

# Equation form of an Exponential Function:

$$y = a(b)^x \quad \text{OR} \quad y = a \cdot b^x$$

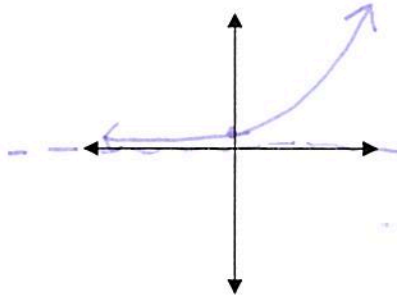
$a =$  Initial       $b =$  growth/decay rate       $x =$  time

What is the difference between exponential growth and exponential decay?

## Exponential Growth

Domain:  $\mathbb{R}$   
 Range:  $y > 0$   
 H. Asymptote:  $y = 0$   
 Critical Point:  $(0, 1)$

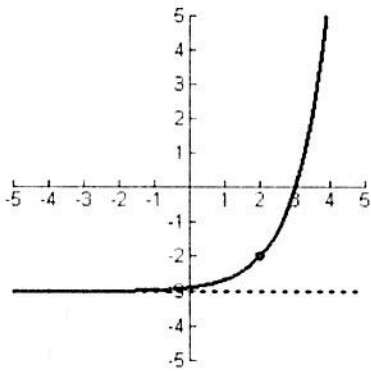
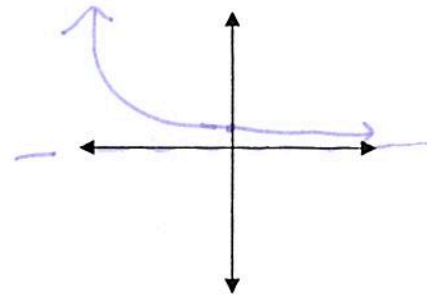
## Graph



## Exponential Decay

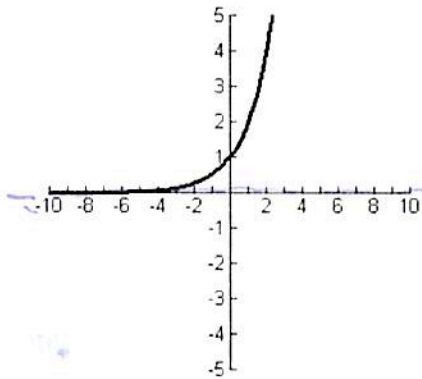
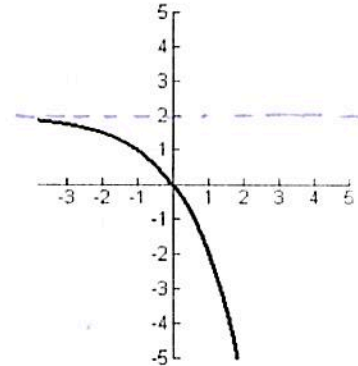
Domain:  $\mathbb{R}$   
 Range:  $y > 0$   
 H. Asymptote:  $y = 0$   
 Critical Point:  $(0, 1)$

## Graph



D:  $\mathbb{R}$   
 R:  $y > -3$   
 Asymptote:  $y = -3$   
 Critical pt:  $(2, -2)$

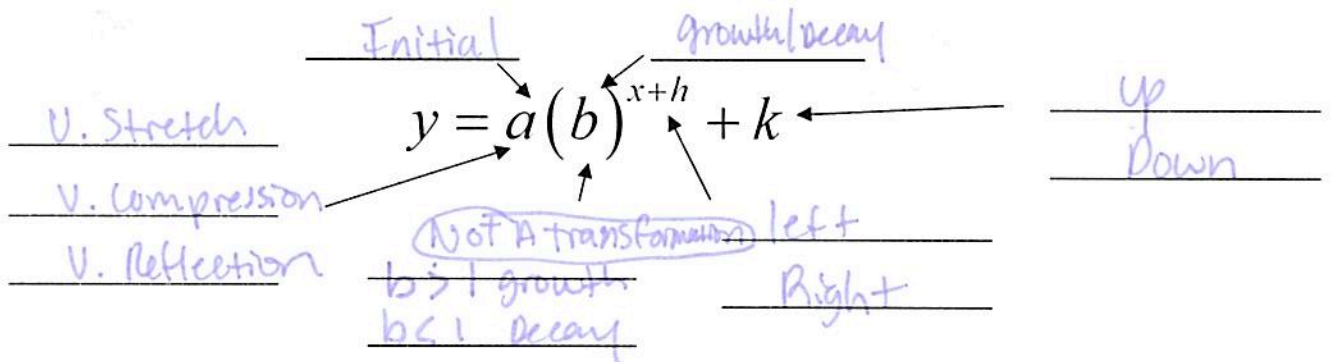
Domain:  $\mathbb{R}$   
 Range:  $y < 2$   
 Asymptote:  $y = 2$



D:  $\mathbb{R}$   
 R:  $y > 0$   
 $y = 0$   
 CP:  $(0, 1)$

Domain:  
 Range:  
 Asymptote:

## Summary



## Transformations of Exponentials

### Explain

Explain the transformations which will take place in each of the situations, state the domain and range AND show how each equation would be input into the calculator.

<p>1. <math>y = (2)^x</math></p> <p>Growth/Decay:</p> <p>Transformations: <i>None</i></p> <p>Critical Point: <math>(0, 1)</math></p> <p>Asymptote: <math>y = 0</math></p> <p>Domain: <math>\mathbb{R}</math></p> <p>Range: <math>y &gt; 0</math></p>	<p>2. <math>y = 2^x + 2</math></p> <p>Growth/Decay:</p> <p>Transformations: <i>up 2</i></p> <p>Critical Point: <math>(0, 3)</math></p> <p>Asymptote: <math>y = 2</math></p> <p>Domain: <math>\mathbb{R}</math></p> <p>Range: <math>y &gt; 2</math></p>	<p>5. <math>y = 2^{x-1} - 2</math></p> <p>Growth/Decay: <i>Right 1; Down 2</i></p> <p>Transformations: <math>\downarrow</math></p> <p>Critical Point: <math>(1, -1)</math> <math>\begin{matrix} (0, 1) \\ +1 &amp; -2 \\ \hline 1, -1 \end{matrix}</math></p> <p>Asymptote: <math>y = -2</math></p> <p>Domain: <math>\mathbb{R}</math></p> <p>Range: <math>y &gt; -2</math></p>
<p>6. <math>y = 3(2)^x</math></p> <p>Growth/Decay:</p> <p>Transformations: <i>V. Stretch</i></p> <p>Critical Point: <math>(0, 3)</math></p> <p>Asymptote: <math>y = 0</math></p> <p>Domain: <math>\mathbb{R}</math></p> <p>Range: <math>y &gt; 0</math></p>	<p>9. <math>y = 4\left(\frac{1}{2}\right)^{x+3}</math></p> <p>Growth/Decay:</p> <p>Transformations: <i>V. Stretch left + 3</i></p> <p>Critical Point: <math>(-3, 4)</math> <math>\begin{matrix} (0, 1) \\ \times 4 \\ \hline 0, 4 \\ -3 \end{matrix}</math></p> <p>Asymptote: <math>y = 0</math></p> <p>Domain: <math>\mathbb{R}</math></p> <p>Range: <math>y &gt; 0</math></p>	<p>11. <math>y = -3(2)^x</math></p> <p>Growth/Decay:</p> <p>Transformations: <i>V. Reflection V. Stretch</i></p> <p>Critical Point: <math>(0, -3)</math> <math>\begin{matrix} (0, 1) \\ 0, -1 \\ \hline 0, -3 \end{matrix}</math></p> <p>Asymptote: <math>y = 0</math></p> <p>Domain: <math>\mathbb{R}</math></p> <p>Range: <math>y &lt; 0</math></p>
<p>13. <math>y = 3(2)^{x-1} - 4</math> <math>\leftarrow +1 \rightarrow</math></p> <p>Transformations: <b>left 3 down 4</b></p> <p>New function: <math>y = 3(2)^{x+2} - 4</math></p> <p>Critical Point: <math>(-2, -1)</math> <math>\begin{matrix} (0, 1) \\ (0, 3) \times 3 \\ -2(-2, -1) - 4 \end{matrix}</math></p> <p>Asymptote: <math>y = -4</math></p> <p>Domain: <math>\mathbb{R}</math></p> <p>Range: <math>y &gt; -4</math></p>	<p>14. <math>y = 4(2)^{x+3} + 2</math></p> <p>Transformations: <b>reflect over x-axis up 5</b></p> <p>New function: <math>y = -4(2)^{x+3} - 2</math> <math>y = -4(2)^{x+3} + 3</math></p> <p>Critical Point: <math>(-3, -1)</math> <math>\begin{matrix} (0, 1) \\ 0, -4 \\ -3, -4 \\ -3, -1 \end{matrix}</math></p> <p>Asymptote: <math>y = 3</math></p> <p>Domain: <math>\mathbb{R}</math></p> <p>Range: <math>y &lt; 3</math></p>	<p>15. <math>y = 5(2)^{x-1} - 2</math></p> <p>Transformations: <b>right 4 up 2</b></p> <p>New function: <math>y = 5(2)^{x+5} - 2</math></p> <p>Critical Point: <math>(5, 5)</math> <math>\begin{matrix} (0, 1) \\ 0, 5 \\ +5 \\ \hline 5, 5 \end{matrix}</math></p> <p>Asymptote: <math>y = 0</math></p> <p>Domain: <math>\mathbb{R}</math></p> <p>Range: <math>y &gt; 0</math></p>



## Transformations of Exponentials

### Evaluate (Assignment)

Explain the transformations which will take place in each of the situations, state the domain and range.

<p>1. <math>y = 3(2)^{x+2}</math></p> <p><u>Growth/Decay</u>:</p> <p>Transformations: V. Stretch left 2</p> <p>Critical Point: <math>(-2, 3)</math></p> <p>Asymptote: <math>y = 0</math></p> <p>Domain: <math>\mathbb{R}</math></p> <p>Range: <math>y &gt; 0</math></p> <div style="margin-left: 100px;"> <math display="block">\begin{array}{r} 0,1 \\ \times 3 \\ \hline 0,3 \\ -2 \\ \hline -2,3 \end{array}</math> </div>	<p>2. <math>y = 5(2)^x + 2</math></p> <p><u>Growth/Decay</u>:</p> <p>Transformations: V. Stretch up 2</p> <p>Critical Point: <math>(0, 7)</math></p> <p>Asymptote: <math>y = 2</math></p> <p>Domain: <math>\mathbb{R}</math></p> <p>Range: <math>y &gt; 2</math></p> <div style="margin-left: 100px;"> <math display="block">\begin{array}{r} 0,1 \\ \times 5 \\ \hline 0,5 \\ +2 \\ \hline 0,7 \end{array}</math> </div>	<p>3. <math>y = \left(\frac{1}{2}\right)^{x+1} + 3</math></p> <p><u>Growth/Decay</u>:</p> <p>Transformations: left 1 up 3</p> <p>Critical Point: <math>(-1, 4)</math></p> <p>Asymptote: <math>y = 3</math></p> <p>Domain: <math>\mathbb{R}</math></p> <p>Range: <math>y &gt; 3</math></p> <div style="margin-left: 100px;"> <math display="block">\begin{array}{r} 0,1 \\ -1 + 3 \\ \hline -1,4 \end{array}</math> </div>
<p>4. <math>y = 1(3)^{x-2} - 1</math></p> <p><u>Growth/Decay</u>:</p> <p>Transformations: Right 2 down 1</p> <p>Critical Point: <math>(2, 0)</math></p> <p>Asymptote: <math>y = -1</math></p> <p>Domain: <math>\mathbb{R}</math></p> <p>Range: <math>y &gt; -1</math></p> <div style="margin-left: 100px;"> <math display="block">\begin{array}{r} (0,1) \\ +2 -1 \\ \hline 2,0 \end{array}</math> </div>	<p>5. <math>y = 2(1.1)^{x-2} - 1</math></p> <p><u>Growth/Decay</u>:</p> <p>Transformations: V. Stretch Down 1 Right 2</p> <p>Critical Point: <math>(2, 1)</math></p> <p>Asymptote: <math>y = -1</math></p> <p>Domain: <math>\mathbb{R}</math></p> <p>Range: <math>y &gt; -1</math></p> <div style="margin-left: 100px;"> <math display="block">\begin{array}{r} (0,1) \\ \times 2 (0,2) \\ +2 -1 \\ \hline 2,1 \end{array}</math> </div>	<p>6. <math>y = -3(2)^{x-1}</math></p> <p><u>Growth/Decay</u>:</p> <p>Transformations: V. Reflect V. Stretch Right 1</p> <p>Critical Point: <math>(1, -3)</math></p> <p>Asymptote: <math>y = 0</math></p> <p>Domain: <math>\mathbb{R}</math></p> <p>Range: <math>y &lt; 0</math></p> <div style="margin-left: 100px;"> <math display="block">\begin{array}{r} 0,1 \\ \times 3 (0, -3) \\ +1 \\ \hline 1, -3 \end{array}</math> </div>

<p>7. <math>f(x) = 6(2)^{x-3} + 3</math></p> <p>Transformations:</p> <p style="text-align: center;"><i>3 right</i> <i>1 up</i></p> <p>New function:</p> <p style="text-align: center;"><math>g(x) = 6(2)^{x-6} + 4</math></p> <p>Critical Point: <math>(6, 10)</math></p> <p>Asymptote: <math>y = 4</math></p> <p>Domain: <math>\mathbb{R}</math></p> <p>Range: <math>y &gt; 4</math></p>	<p>8. <math>f(x) = 5(2)^{x+2} - 4</math></p> <p>Transformations:</p> <p style="text-align: center;"><i>2 left</i> <i>4 up</i></p> <p>New function:</p> <p style="text-align: center;"><math>g(x) = 5(2)^{x+4}</math></p> <p>Critical Point: <math>(-4, 5)</math></p> <p>Asymptote: <math>y = 0</math></p> <p>Domain: <math>\mathbb{R}</math></p> <p>Range: <math>y &gt; 0</math></p>	<p>9. <math>f(x) = 2(0.5)^{x-1} + 1</math></p> <p>Transformations:</p> <p style="text-align: center;"><i>1 right</i> <i>1 down</i></p> <p>New function:</p> <p style="text-align: center;"><math>g(x) = 2\left(\frac{1}{2}\right)^{x-2}</math></p> <p>Critical Point: <math>(2, 2)</math></p> <p>Asymptote: <math>y = 0</math></p> <p>Domain: <math>\mathbb{R}</math></p> <p>Range: <math>y &gt; 0</math></p>
<p>10. <math>f(x) = 2^x</math></p> <p>Transformations:</p> <p style="text-align: center;"><i>6 right</i> <i>2 up</i> <i>vertical stretch of -3</i></p> <p>New function:</p> <p style="text-align: center;"><math>g(x) = -3(2)^{x-6} + 2</math></p> <p>Critical Point: <math>(6, -1)</math></p> <p>Asymptote: <math>y = 2</math></p> <p>Domain: <math>\mathbb{R}</math></p> <p>Range: <math>y &lt; 2</math></p>	<p>11. <math>f(x) = 5(2)^{x+2} - 4</math></p> <p>Transformations:</p> <p style="text-align: center;"><i>2 left</i> <i>1 down</i></p> <p>New function:</p> <p style="text-align: center;"><math>g(x) = 5(2)^{x+4} - 5</math></p> <p>Critical Point: <math>(-4, 0)</math></p> <p>Asymptote: <math>y = -5</math></p> <p>Domain: <math>\mathbb{R}</math></p> <p>Range: <math>y &gt; -5</math></p>	<p>12. <math>f(x) = 2(3)^{x+2} - 4</math></p> <p>Transformations:</p> <p style="text-align: center;"><i>Reflect across the x-axis</i> <i>2 right</i></p> <p>New function:</p> <p style="text-align: center;"><math>g(x) = -2(3)^x + 4</math></p> <p>Critical Point: <math>(0, 2)</math></p> <p>Asymptote: <math>y = 4</math></p> <p>Domain: <math>\mathbb{R}</math></p> <p>Range: <math>y &lt; 4</math></p>

*Make sure to negate both y transforms*