

Sec. 5.2: Polynomial Division

Long Division Steps:

1. Divide the first term of the dividend by the first term of the divisor.
2. Write the result from step 1 in the quotient and use it to multiply the divisor.
3. Subtract the product from the dividend.
4. Repeat steps 1-3 using the difference from step 3 as the new dividend.

Example: Find the quotient. $(x^3 + 3x^2 + 3x + 2) \div (x^2 + x + 1)$

Dividend = $(x^3 + 3x^2 + 3x + 2)$

Divisor = $(x^2 + x + 1)$

$$\begin{array}{r}
 \overline{) x^3 + 3x^2 + 3x + 2} \\
 \underline{-(x^3 + x^2 + x)} \\
 2x^2 + 2x + 2 \\
 \underline{-(2x^2 + 2x + 2)} \\
 0 \leftarrow \text{remainder}
 \end{array}$$

Use long division.

1 $(x^2 + 5x - 14) \div (x - 2)$

$$\begin{array}{r}
 \overline{) x^2 + 5x - 14} \\
 \underline{-(x^2 - 2x)} \downarrow \\
 7x - 14 \\
 \underline{-(7x - 14)} \\
 0 \checkmark
 \end{array}$$

$x + 7$

2 $(2x^2 - 17x - 38) \div (2x + 3)$

$$\begin{array}{r}
 \overline{) 2x^2 - 17x - 38} \\
 \underline{-(2x^2 + 3x)} \downarrow \\
 -20x - 38 \\
 \underline{-(-20x - 30)} \\
 -8
 \end{array}$$

$x - 10 + \frac{-8}{2x + 3}$

3. $(x^4 - 7x^2 + 9x - 10) \div (x - 2)$

$$\begin{array}{r}
 x^3 + 2x^2 - 3x + 3 \\
 x - 2 \overline{) x^4 + 0x^3 - 7x^2 + 9x - 10} \\
 \underline{-(x^4 - 2x^3)} \downarrow \\
 2x^3 - 7x^2 + 9x - 10 \\
 \underline{-(2x^3 - 4x^2)} \downarrow \\
 -3x^2 + 9x - 10 \\
 \underline{-(-3x^2 + 6x)} \downarrow \\
 3x - 10 \\
 \underline{-(3x - 6)} \\
 -4
 \end{array}$$

$x^3 + 2x^2 - 3x + 3 + \frac{-4}{x-2}$

4. $(x^3 - x^2 - 21x + 45) \div (x^2 + 2x - 15)$

$$\begin{array}{r}
 -3 \\
 x^2 + 2x - 15 \overline{) x^3 - x^2 - 21x + 45} \\
 \underline{-(x^3 + 2x^2 - 15x)} \downarrow \\
 -3x^2 - 6x + 45 \\
 \underline{-(x^2 + 2x - 15)} \\
 -8x + 60 \\
 \underline{-(-8x + 60)} \\
 0
 \end{array}$$

$x^2 + 2x - 15 + \frac{-8x + 60}{x^2 + 2x - 15}$

$x - 3$

5. $(8x^3 + 5x^2 + 12x + 10) \div (x^2 + 3)$

$$\begin{array}{r}
 8x + 5 \\
 x^2 + 0x + 3 \overline{) 8x^3 + 5x^2 + 12x + 10} \\
 \underline{-(8x^3 + 0x^2 + 24x)} \downarrow \\
 5x^2 - 12x + 10 \\
 \underline{-(5x^2 + 0x + 15)} \\
 -12x - 5
 \end{array}$$

$8x + 5 + \frac{-12x - 5}{x^2 + 3}$

6. $(4x^4 + 2x^3 - 9x + 12) \div (x^2 - 2x)$

$$\begin{array}{r}
 4x^2 + 10x + 20 \\
 x^2 - 2x \overline{) 4x^4 + 2x^3 + 0x^2 - 9x + 12} \\
 \underline{-(4x^4 - 8x^3)} \\
 10x^3 + 8x^2 - 9x + 12 \\
 \underline{-(10x^3 - 20x^2)} \downarrow \\
 20x^2 - 9x + 12 \\
 \underline{-(20x^2 - 40x)} \\
 31x + 12
 \end{array}$$

$4x^2 + 10x + 20 + \frac{31x + 12}{x^2 - 2x}$

Synthetic Division Steps:

1. Write the coefficients of the polynomial and then write the value of r on the left. Write the first coefficient below the line.
2. Multiply the r -value by the number below the line, and write the product below the next coefficient.
3. Write the sum (not the difference) below the line. Multiply r by the number below the line and write the product below the next coefficient.
4. Write the sum (not the difference) below the line. Repeat steps 1-3 as needed.

Note: Synthetic Division can only be used on linear divisors (i.e. in the form $x - r$). If the divisor is in any other form, Long Division must be used.

Use synthetic division.

7. $(x^2 + 5x - 6) \div (x - 1)$

$$\begin{array}{r|rrrr} & 1 & 5 & -6 & \\ & \downarrow & & & \\ & 1 & 6 & & \\ \hline & x & & 6 & \end{array}$$

$$\boxed{x + 6}$$

8. $(x^3 + x + 30) \div (x + 3)$

$$\begin{array}{r|rrrr} & 1 & 0 & 1 & 30 \\ & \downarrow & -3 & 9 & -30 \\ & 1 & -3 & 10 & \\ \hline & x^2 & x & & 10 \end{array}$$

$$\boxed{x^2 - 3x + 10}$$

9. $(2x^4 - 11x^3 + 15x^2 + 6x - 18) \div (x - 3)$

$$\begin{array}{r} 3 \overline{) 2 \ -11 \ 15 \ 6 \ -18} \\ \underline{\downarrow 6 \ -15 \ 0 \ 18} \\ 2 \ -5 \ 0 \ 6 \ \checkmark \\ x^3 \quad x^2 \quad x \quad \# \end{array}$$

$$2x^3 - 5x^2 + 6$$

10. $(x^2 - 2x - 48) \div (x + 5)$

$$\begin{array}{r} -5 \overline{) 1 \ -2 \ -48} \\ \underline{\downarrow -5 \ 35} \\ 1 \ -7 \ \boxed{-13} \\ x \quad \# \end{array}$$

$$x - 7 + \frac{-13}{x+5}$$

11. $(x^4 - 7x^2 + 9x - 10) \div (x - 2)$

$$\begin{array}{r} \downarrow \\ 2 \overline{) 1 \ 0 \ -7 \ 9 \ -10} \\ \underline{\downarrow 2 \ 4 \ -6 \ 6} \\ 1 \ 2 \ -3 \ 3 \ \boxed{-4} \\ x^3 \quad x^2 \quad x \quad \# \end{array}$$

$$x^3 + 2x^2 - 3x + 3 + \frac{-4}{x-2}$$

12. $(3x^3 - 16x^2 - 103x + 36) \div (x + 4)$

$$\begin{array}{r} -4 \overline{) 3 \ -16 \ -103 \ 36} \\ \underline{\downarrow -12 \ 112 \ -36} \\ 3 \ -28 \ 9 \ \boxed{0} \\ x^2 \quad x \quad \# \end{array}$$

$$3x^2 - 28x + 9$$

$$\begin{array}{r} 328 \\ 4 \\ \hline 112 \end{array}$$

Use long or synthetic division.

1. $(x^3 - 10x^2 + 19x + 30) \div (x - 6)$

$$\begin{array}{r|rrrr}
 6 & 1 & -10 & 19 & 30 \\
 & \downarrow & 6 & -24 & -30 \\
 \hline
 & 1 & -4 & -5 & 0 \\
 & x^2 & x & \# & \text{R}
 \end{array}$$

$x^2 - 4x - 5$

2. $(x^3 - 4x + 6) \div (x + 3)$

$$\begin{array}{r|rrrr}
 -3 & 1 & 0 & -4 & 6 \\
 & \downarrow & -3 & 9 & -15 \\
 \hline
 & 1 & -3 & 5 & -9 \\
 & x^2 & x & \# & \text{R}
 \end{array}$$

$x^2 - 3x + 5 + \frac{-9}{x+3}$

3. $(4x^4 - 15x^2 - 4) \div (x + 2)$

$$\begin{array}{r|rrrrr}
 -2 & 4 & 0 & -15 & 0 & -4 \\
 & \downarrow & -8 & 16 & -2 & 4 \\
 \hline
 & 4 & -8 & 1 & -2 & 0 \\
 & x^3 & x^2 & x & \# & \text{R}
 \end{array}$$

$4x^3 - 8x^2 + x - 2$

4. $(3x^3 + 11x^2 + 4x + 1) \div (x^2 + x)$

$$\begin{array}{r}
 3x+8 \\
 x^2+x \overline{) 3x^3+11x^2+4x+1} \\
 \underline{-(3x^3+3x^2)} \downarrow \\
 8x^2+4x+1 \\
 \underline{-(8x^2+8x)} \downarrow \\
 -4x+1 \text{R}
 \end{array}$$

$3x + 8 + \frac{-4x+1}{x^2+x}$

5. $(x^4 - 5x^3 - 8x^2 + 13x - 12) \div (x - 6)$

$$\begin{array}{r|rrrrr} 6 & 1 & -5 & -8 & 13 & -12 \\ & \downarrow & 6 & 6 & -12 & 6 \\ \hline & 1 & 1 & -2 & 1 & \underline{-6} \text{ R} \\ & x^3 & x^2 & x & \# & \end{array}$$

$$x^3 + x^2 - 2x + 1 + \frac{-6}{x-6}$$

6. $(3x^3 + 34x^2 + 72x - 64) \div (3x - 2)$

$$\begin{array}{r} x^2 + 12x + 32 \\ 3x-2 \overline{) 3x^3 + 34x^2 + 72x - 64} \\ \underline{-(3x^3 - 2x^2)} \quad \downarrow \\ 36x^2 + 72x - 64 \\ \underline{-(36x^2 - 24x)} \quad \downarrow \\ 96x - 64 \\ \underline{-(96x - 64)} \\ 0 \checkmark \end{array}$$

$$\boxed{x^2 + 12x + 32}$$

7. When the polynomial $x^4 + 4x^3 + 5x^2 + 16x - 16$ is divided by $x + 4$, the quotient is $x^3 + Ax + B$. What are the values of A and B?

$$\begin{array}{r|rrrrr} -4 & 1 & 4 & 5 & 16 & -16 \\ & \downarrow & -4 & -9 & -20 & 16 \\ \hline & 1 & 0 & -4 & -4 & \underline{0} \text{ R} \\ & x^3 & x^2 & x & \# & \end{array}$$

$$\boxed{x^3 + 5x - 4}$$

$$x^3 + Ax + B$$

$$\boxed{A = 5 \quad B = -4}$$