PAP Algebra 2
Annual Percentage Rates:

$a(b)^x$

Now that you are experts at finding percents, let's apply that knowledge to comparing credit cards. Card A has an APR of 10%. Card B has an APR of 25%. Card C has an APR of 50% (this is realistic for illegal enterprises, and payday loans.) Let's say you start off with a $100 balance on all cards, and you are not making any payments (cause you're bad like that). How much will you owe in each year?

1. Complete the table.

Year	Card A (10%)	Card B (25%)	Card C (50%)
0	100	100	100
1	$100(1+.10)^{x\ 100}$	$100(1+.25)^x$	$100(1+.50)^x$
3			
5			
10			
20			
x (*type into Y1*)	$100(1+.10)^x$	$100(1+.25)^x$	$100(1+.5)^x$

2. Create a graph in your calculator that shows the balances of all 3 credit cards.
   Use the following window

Xmin: 0	Ymin: 0
Xmax: 12	Ymax: 950
Xscl: 1	Yscl: 50

3. Using your calculator, find the year in which the amount you owe doubles for each card.

Card A 8 years Card B 4 years Card C 2 years

4. What would be the interest rate on a card that, on an initial balance of $100, owed $228.78 after 5 years, and $523.38 after 10 years?

STAT 1:edit L1:x L2:y
STAT → 0: ExpReg

0	100
5	228.78
10	583.38

$y = 100(1.18)^x$

In reality, interest on a credit card isn't only charged once a year. Let's investigate how it's really calculated. Suppose you charge $1,000 on a card that charges 12% APR.

5. Complete the Table:

Frequency of Calculation	#of Times Interest is Calculated per Year	Interest Rate per Period at 12% APR	Expression for Balance after x years	Account Balance After 5 Years
Annual	1	.12	$1{,}000(1+\frac{.12}{1})^{1t}$	1762.34
Semi-Annual	2	$\frac{.12}{2}$	$1000(1+\frac{.12}{2})^{2t}$	1790.85
Quarterly	4	$\frac{.12}{4}$	$1000(1+\frac{.12}{4})^{4t}$	1806.11
Monthly	12	$\frac{.12}{12}$	$1000(1+\frac{.12}{12})^{12t}$	~~[illegible]~~ 1816.69
Weekly	52	$\frac{.12}{52}$	$1000(1+\frac{.12}{52})^{52t}$	1820.86

6. Try to write an expression using only variables, for the account balance *A*, after *t* years, on an initial charge *P*, at an annual percentage rate *r*, calculated *n* times a year.

$$y = 1000(1+\frac{.12}{n})^{nt}$$

7. **A dastardly accountant at the credit card company notices that if they charge interest more frequently, they will earn more money. (Sure, not much, but if they do it to everybody...) How frequently will they have to charge 12% interest in order for you to owe $1128 after one year?** Complete the table.

Frequency of Calculation	# of Times Interest is Calculated per Year	Interest Rate Each Period	Expression for Balance on $1000 after 1 year	Account Balance on $1000 after 1 Year
Daily	365	.12/360	$1000(1+\frac{.12}{365})^{365t}$	1127.48
Hourly	8760	.12/8760	$1000(1+\frac{.12}{8760})^{8760t}$	1127.49
Minute-ly	525600	.12/525600	$1000(1+\frac{.12}{525600})^{525600t}$	1127.5

What?! There doesn't seem to be a way to even reach \$1127.50! What if we could charge interest *all the time*? Surely, if we could compound the interest infinite times per year, we could make infinite dollars. Sounds pretty amazing.

. To investigate, let's consider a very simple example. 100% APR, on a \$1 charge over 1 year for more and more frequent compounding. Show at least four decimal places.

# of times compounded in the year (n)	10	1000	10,000	100,000	1,000,000
Substitute: $\left(1+\frac{1}{n}\right)^n$	$\left(1+\frac{1}{10}\right)^{10}$	$\left(1+\frac{1}{1000}\right)^{1000}$	$\left(1+\frac{1}{10000}\right)^{10000}$	$\left(1+\frac{1}{100000}\right)^{100000}$	$\left(1+\frac{1}{1000000}\right)^{1000000}$
Evaluate to 4 places	2.5931	2.716	2.718	2.718	2.718

Sorry, accountant, but your little scheme just got shut down!!! There is a *limit* to how much interest they can earn, despite very frequent compounding.

9. As n approaches ∞, this value is called $\boxed{e}$.
Approximate e to three decimal places: 2.718

That is very unfortunate for the accountant. It seems that we can't earn \$$\infty$. Darn!!! The constant e is used in situations that involve <u>*continuous growth*</u> such as...

- Growth of the body or any living organism
- Population growth (of very large populations)
- Half life of an organic compound (carbon dating)

Continuous compound growth is modeled with:

$$\boxed{\mathbf{A = Pe^{rt}}}$$

...where P represents the initial value, r represents the growth rate, t represents units of time, and A represents the ending value.

Apply your knowledge:

1. You owe \$200 on a debt with 16% APR, compounded monthly. How much will you owe after 2 years? Try to do this by evaluating a single expression. Write down the expression as well as the answer.

Expression you evaluated: $200\left(1+\frac{.16}{12}\right)^{12t}$ Balance: \$274.84

2. Suppose you invest $1,050 at an annual interest rate of 5.5% compounded continuously. How much money, to the nearest dollar, will you have in the account after five years? (Financial institutions will not, in general, offer interest rates that are compounded continuously)

$1{,}050e^{.055t} = \boxed{1382.36}$

3. According to statistical surveys, the annual growth rate in the world population in recent years is about 1.7%. There were about 5.3 billion people living on this planet in 1990.

Unlike bank accounts, the compounding of population growth does not take place annually or quarterly. It's going on all the time. Every second of every hour, people are being born and others are dying. Thus, the growth rate (the overall effect of all of those births and deaths) can be viewed as a ***continuous process***.

If the same growth rate continues, how many people will inhabit the Earth by 2012?

$t = 22$ years

$5.3e^{.017t} = \boxed{7.7 \text{ Billion}}$

4. You deposit $1,000 in a college fund that pays 6.1% interest compounded quarterly. Find the account balance after 5 years. Write down the expression as well as the answer.

$1000\left(1 + \frac{.061}{4}\right)^{4t} = \boxed{1353.50}$

5. Suppose you have $1,500 in a savings account that pays 4.7% annual interest. Find the account balance after 25 years with the interest compounded semi-annually.

$1{,}500\left(1 + \frac{.047}{1}\right)^{1t} = \boxed{4728.88}$

6. You deposit $4,500 into an account earning 3.6%, compounded quarterly. How much will be in the account after 12 years?

$4{,}500\left(1 + \frac{.036}{4}\right)^{4t} = \boxed{6918.13}$

12.5 Exponential Application Homework **Name:** key
PAP Algebra 2

1. On the day of a child's birth, a deposit of $25,000 is made in a trust fund that pays 6.25% interest. Determine the balance in this account on the child's 26th birthday if the interest is compounded:

6.25

a) quarterly

$25000\left(1+\frac{.0625}{4}\right)^{4t}$

$\boxed{125375.25}$

b) monthly

$25000\left(1+\frac{.0625}{12}\right)^{12t}$

$\boxed{126426.19}$

c) continuously

$25000\, e^{.0625t}$

$\boxed{126960.48}$

2. What is the initial value of a Samsung Galaxy phone if it depreciates at a value of 2.5% per week if after 57 weeks it was worth $124?

$124 = a(1-.025)^{57}$

$\boxed{a = 524.99 \text{ or } 525}$

3. How much money would you need to deposit today at 9% annual interest compounded monthly to have $12,000 in the account after 6 years?

$12{,}000 = a\left(1+\frac{.09}{12}\right)^{12 \cdot 6}$

$\boxed{a = 7007}$

For #'s 4-5, find the balance in each account for the different compounding periods.

4. $15,000 principal earning 3.5% interest after 4 years

a) Annually

$15{,}000(1+.035)^{4}$

$\boxed{1721.28}$

b) Semi-annually

$15{,}000\left(1+\frac{.035}{2}\right)^{2x}$

$\boxed{1723.32}$

c) Quarterly

$15000\left(1+\frac{.035}{4}\right)^{4t}$

$\boxed{1724.36}$

d) Monthly

$15000\left(1+\frac{.035}{12}\right)$

$\boxed{1725.06}$

5. The half-life of a radioactive isotope is the time it takes for half the material to become inert. Starting with 100g of radioactive material, find the number of grams still radioactive after 10 days if the half-life is:

a)10 days

$100(1/2)^{\frac{x}{10}}$

$\boxed{50}$

b) 1 day

$100(1/2)^{\frac{1}{10}}$

$\boxed{93.3}$

c) 20 days

$100(1/2)^{\frac{20}{10}}$

$\boxed{25}$

d) 12 hours

12 → 1/2 Da

$100(1/2)^{\frac{12}{10}}$

$\boxed{96.59}$

$\boxed{\cancel{93.303}}$

6. A student wants to have $10,000 for college 5 years from now. How much should she put into an account that earns 4.1% annual interest compounded continuously?

$10{,}000 = a(\cancel{e})^{.041t}$

$10{,}000 = ae^{.041(5)}$

$\boxed{a = 8{,}146.47}$

7. How long would it take to double your principal at an annual interest rate of 8% compounded continuously?

$8 = 4e^{.08t}$

$2 = e^{.08t}$

$\frac{\ln 2}{.08} = \frac{.08t}{.08}$

$\boxed{t = 8.66}$