

Inverse Variation $y = \frac{k}{x}$

$$k = x \cdot y$$

$$xy = xy$$



Timeline Activity

# of People			
Time			

As x \downarrow , y \uparrow or as x \uparrow , y \downarrow

Equation:

1. If $y = 27$ when $x = 12$. $k = xy \quad 27 \cdot 12 = 324$

a) Find the inverse equation. $y = \frac{k}{x} \rightarrow y = \frac{324}{x}$

b) Find x when $y = -12$
 $324 = -12x$
 $x = -27$

2. The grade you earn in math varies inversely with the number of minutes per night you watch TV. If you watch 90 minutes per night, you get a 60 in math.

a) Find the constant of variation.

$$90(60) = 5400$$

b) How much TV can you watch if you want to make a 70? $5400 = x \cdot 70$ $x = 77.14 \text{ mins!}$

c) If you watch 75 min of TV a night, what grade will you make? $5400 = 75 \cdot y$
 $y = 72$

d) What is the maximum amount of television you can watch and still make a 100?

$$5400 = 100x$$

$$x = 54 \text{ mins}$$

3. The length of a violin string varies inversely with the frequency of its vibrations. A violin string 10 inches long vibrates at a frequency of 512 cycles per second. Let x = length in inches, and y = the frequency per second.

a) Find the frequency, in cycles per second, of the 8-inch string.

$$10 \cdot 512 = 8y$$

$$y = 640$$

b) What is the length of a string with a frequency of 320 cycles per second?

$$5120 = 320x$$

$$x = 16$$

4. You are driving from Dallas to Austin for a college visit. Remember the distance formula $D = rt$. $t = \frac{D}{r}$



a) What happens to the amount of time it takes to make the trip if you increase your speed?

Time Decreases

b) What is the constant? Distance:
 $t = \frac{D}{r}$

c) If your rate is 55 mph, the trip takes 2.5 hours, how can we find r if $t = 1.25$ hours.

$$55(2.5) = r(1.25)$$

$$137.5 = 1.25r$$

$$r = 110$$

Practice Problems:

1. If x and y vary inversely and $x = 1$ when $y = 11$, find x when $y = 5.5$

$$1 \cdot 11 = x \cdot 5.5$$

$$11 = 5.5x$$

$$x = 2$$

2. If x and y vary inversely and $x = 2.5$ when $y = 100$ find x when $y = 25$

$$2.5(100) = x \cdot 25$$

$$x = 10$$

3. Heart rates and life spans of most mammals are inversely related. A cat lives for about 15.2 years on average and has a heart rate of 126 beats per minute.

a. What is the constant of variation? And what the inverse equation?

$$15.2(126) = 1915.2$$

$$k = 1915.2$$

$$y = \frac{1915.2}{x}$$

b. A hamster has a heart rate of about 634 beats per minute. About how long will a hamster live?

$$1915.2 = 634x$$

$$x \approx 3.02$$

c. An elephant lives for about 70 years. About how many times per minute does an elephant's heart beat?

$$1915.2 = 70y$$

$$y \approx 27.36$$

4. Two gears are used to operate a machine. Gear A has 60 teeth and Gear B has 45 teeth. The speed at which you turn Gear A is 5400 rpm. The number of teeth and speed in rpm are inversely related.

a. What is the constant of variation? And what the inverse equation?

$$k = 60(5400)$$

$$k = 324000$$

$$y = \frac{324000}{x}$$

b. At what speed will Gear B turn?

$$324000 = 45y$$

$$y = 7200$$

PAP Algebra 2
Chapter 9 Test Review

Name Key

Simplify the following expressions.

1. $\frac{x-2}{x^2+7x-18} \div \frac{x^3-6x^2-27x}{x^2+12x+27}$ Keep change flip

$$\frac{\cancel{x-2}}{\cancel{(x+9)}(x-2)} \cdot \frac{\cancel{(x+9)}(x+3)}{x(x^2-6x-27)} = \frac{1}{x(x-9)}$$

2. $\frac{1}{x^2-x-2} - \frac{x}{x^2-5x+6}$ LCD: $(x-2)(x+1)(x-3)$

$$\frac{1}{(x-2)(x+1)} - \frac{x}{(x-2)(x-3)}$$

$$\frac{1}{(x-2)(x+1)} \cdot \frac{x-3}{x-3} = \frac{x-3}{(x-2)(x+1)(x-3)}$$

$$\frac{-x}{(x-2)(x-3)} \cdot \frac{x+1}{x+1} = \frac{-x(x+1)}{(x-2)(x+1)(x-3)}$$

$$\frac{x-3}{(x-2)(x+1)(x-3)} + \frac{-x^2-x}{(x-2)(x+1)(x-3)}$$

$$\frac{-x^2-3}{(x-2)(x+1)(x-3)}$$

3. $\frac{x+5}{4x-16} \cdot \frac{2x^2-32}{x^2-25}$

$$\frac{\cancel{2}(x+4)}{\cancel{4}(x-4)} \cdot \frac{\cancel{2}(x^2-16)}{(x+5)(x-5)}$$

$$\frac{1}{2} \cdot \frac{2(x+4)}{x-5} = \frac{x+4}{x-5}$$

4. $\frac{3x^2+13x+4}{x^2-4} \div \frac{4x+16}{x+2}$ Keep change flip

$$\frac{(3x+1)(x+4)}{(x+2)(x-2)} \cdot \frac{x+2}{4(x+4)} = \frac{3x+1}{4(x-2)}$$

5. $\frac{x+4}{x^2-4} - \frac{15}{x-2}$ LCD: $(x+2)(x-2)$

$$\frac{x+4}{(x+2)(x-2)} - \frac{15}{x-2}$$

$$\frac{x+4}{(x+2)(x-2)} \cdot \frac{1}{1} = \frac{x+4}{(x+2)(x-2)}$$

$$\frac{-15}{x-2} \cdot \frac{x+2}{x+2} = \frac{-15(x+2)}{(x+2)(x-2)}$$

$$\frac{x+4}{(x+2)(x-2)} + \frac{-15x-30}{(x+2)(x-2)}$$

$$\frac{-14x-26}{(x+2)(x-2)}$$

Solving: Identify excluded solutions

$$6. \frac{x}{x-1} + x = \frac{4x-3}{x-1}$$

$$x = \frac{4x-3}{x-1} - \frac{x}{x-1}$$

$$\frac{x}{1} = \frac{3x-3}{x-1}$$

$$3x-3 = x(x-1)$$

$$3x-3 = x^2-x$$

$$\begin{array}{r} -3x+3 \\ -3x+3 \end{array}$$

$$0 = x^2 - 4x + 3$$

$$0 = (x-3)(x-1)$$

$$\boxed{x=3} \quad x \neq 1$$

$$x=1$$

$$7. 1 = \frac{1}{x^2+2x} + \frac{(x-1)(x+2)}{x(x+2)}$$

$$1 = \frac{1}{x(x+2)} + \frac{x^2+x-2}{x(x+2)}$$

$$\frac{1}{1} = \frac{x^2+x-1}{x(x+2)}$$

$$x^2+2x = x^2+x-1$$

$$\begin{array}{r} 2x = x-1 \\ -x \quad -x \end{array}$$

$$\boxed{x = -1}$$

$$x \neq 0, -2$$

$$8. \frac{3x}{x^2-x-20} + \frac{2}{x+4} = \frac{5x}{x-5}$$

$$3x + 2(x-5) = 5x(x+4)$$

$$3x + 2x - 10 = 5x^2 + 20x$$

$$5x - 10 = 5x^2 + 20x$$

$$0 = 5x^2 + 15x + 10$$

$$5(x^2 + 3x + 2)$$

$$5(x+2)(x+1)$$

$$\boxed{x = -2}$$

$$\boxed{x = -1}$$

$$x \neq 5, 4$$

Word Problems

9. The number y of boxes of candy a manufacturer sells each month varies inversely with the price x (in dollars). In one month, the manufacturer sells 800 boxes of candy at a price of \$5 per box. Approximately how many boxes of candy will the manufacturer sell at a price \$7 per box?

$$5(800) = 7y$$

$$4000 = 7y$$

$$y = 571.4$$

$$\boxed{\text{About 572 boxes}}$$

10. A car travels 120 miles in the same amount of time that it takes a truck to travel 100 miles. The car travels 10 miles per hour faster than the truck. Use the model to find the speed of the truck.

$$t = \frac{d}{s}$$

$$\frac{120}{x+10} = \frac{100}{x}$$

$$100(x+10) = 120x$$

$$100x + 1000 = 120x$$

$$1000 = 20x$$

$$\boxed{x = 50 \text{ mph}}$$

Work

11. Bob can paint a fence in 6 hours and Sam can paint a fence in 10 hours. How long would it take both men to paint 3 fences working together?

LCM: ~~60~~ $30x$

$$\left(\frac{1}{6} + \frac{1}{10} = \frac{1}{x}\right) 30x$$

$$\frac{30x}{6} + \frac{30x}{10} = \frac{30x}{x}$$

$$5x + 3x = 30$$

$$8x = 30 \quad x = 3.75 \rightarrow 1 \text{ fence}$$

3 fences
↓
 $3(3.75) = 11.25$

11 hours and 15 mins

12. The equation for the time it takes a boat to travel 40 kilometers round trip going against and then with the current $t = \frac{20}{s-2} + \frac{20}{s+2}$ where s is the boat's average speed (km/h) in still water. Find the total travel time when the average speed of the boat is 18 kilometers per hour.

$$t = \frac{20}{18-2} + \frac{20}{18+2}$$

$$t = \frac{20}{16} + \frac{20}{20}$$

$$t = \frac{20}{16} + 1$$

$$t = \frac{5}{4} + 1$$

LCM: 4

$$t = \frac{5}{4} + \frac{4}{4}$$

$$t = \frac{9}{4}$$

$t = 2.25$ ~~hours~~

2 hrs and 15 min

13. A boat can travel 8 miles an hour in still water. If it can travel 15 miles down a stream in the same time that it can travel 9 miles up the stream, what is the rate of the stream?

X → Rate of Stream

$$\frac{15}{8+x} = \frac{9}{8-x}$$

$$15(8-x) = 9(8+x)$$

$$120 - 15x = 72 + 9x$$

$$\begin{array}{r} 120 - 15x = 72 + 9x \\ 15x \quad \quad 15x \\ \hline 120 = 72 + 24x \end{array}$$

$$\frac{48}{24} = \frac{24x}{24}$$

$x = 2 \text{ mph}$

14. Jamie, Pria and Paul can paint a room together in 2 hours. If Pria does the job alone she can paint the room in 5 hours. If Paul works alone, he can paint the room in 6 hours. If Jamie works alone, how long would it take her to paint the room?

LCM: $30x$

$$\left(\frac{1}{5} + \frac{1}{6} + \frac{1}{x} = \frac{1}{2}\right) 30x$$

$$\frac{30x}{5} + \frac{30x}{6} + \frac{30x}{x} = \frac{30x}{2}$$

$$6x + 5x + 30 = 15x$$

$$11x + 30 = 15x$$

$$30 = 4x$$

$$x = \frac{15}{2} \text{ or } 7.5 \rightarrow 7 \text{ hrs and } 30 \text{ mins}$$

15. The volume V of a gas varies inversely as the pressure P on it. If the volume is 240 cm^3 under pressure of 30 kg/cm^2 , what pressure has to be applied to have a volume of 160 cm^3 ?

$$V \cdot P = V \cdot P$$

$$240(30) = 160P$$

$$7200 = 160P$$

$$P = 45 \text{ kg/cm}^2$$

16. The length of a violin string varies inversely as the frequency of its vibrations. A violin string 14 inches long vibrates at a frequency of 450 cycles per second. Find the frequency of a 12-inch violin string.

$$L \cdot f = L \cdot f$$

$$14(450) = 12f$$

$$6300 = 12f$$

$$f = 525 \text{ cycles per sec.}$$

17. The manufacturer of cell phones has a fixed cost of $\$26,000$, plus a cost of $\$28$ per phone.

Average cost **per** phone equation: $y = \frac{28x + 26,000}{x}$

Range: $(28, 26028)$

Find the average cost per phone if the manufacturer produces 1000 phones.

$$y = \frac{28(1000) + 26,000}{1000}$$

$$y = \$54.00$$

18. ^{work} Suppose one painter can paint the entire house in twelve hours, and the second painter takes eight hours. How long would it take the two painters together to paint the house?

LCD: $48x$ $48x \left(\frac{1}{12} + \frac{1}{8} = \frac{1}{x} \right)$

$$\frac{48x}{12} + \frac{48x}{8} = \frac{48x}{x}$$

$$4x + 6x = 48$$

$$10x = 48$$

$$x = 4.8 \rightarrow 4 \text{ hours and } 48 \text{ mins}$$