

Simplifying Polynomials – Divide (Long and Synthetic Division)

1) $(4x^4 - 15x^2 - 4) \div (x + 2)$

$$\begin{array}{r} -2 \overline{) 4 \ 0 \ -15 \ 0 \ -4} \\ \underline{\downarrow -8 \ 16 \ -2 \ 4} \\ 4 \ -8 \ 1 \ -2 \ \boxed{0} \\ x^3 \ x^2 \ x \ c \ R \end{array}$$

$4x^3 - 8x^2 + x - 2$

2) $(x^5 - 1) \div (x + 1)$

$$\begin{array}{r} -1 \overline{) 1 \ 0 \ 0 \ 0 \ 0 \ -1} \\ \underline{\downarrow -1 \ 1 \ -1 \ 1 \ -1} \\ 1 \ -1 \ 1 \ -1 \ 1 \ \boxed{0} \\ x^5 \ x^4 \ x^3 \ x^2 \ x \ c \ R \end{array}$$

$x^4 - x^3 + x^2 - x + 1$

3) $(3x^3 + 34x^2 + 72x - 64) \div (x^2 + 12x + 32)$

$$\begin{array}{r} 3x - 2 \overline{) 3x^3 + 34x^2 + 72x - 64} \\ \underline{-(3x^3 + 36x^2 + 96x)} \\ -2x^2 + -24x - 64 \\ \underline{-(-2x^2 - 24x - 64)} \\ 0 \end{array}$$

$3x - 2$

4) $(x^4 + 4x^3 - x - 4) \div (x^3 - 1)$

$$\begin{array}{r} x + 4 \overline{) x^4 + 4x^3 + 0x^2 - x - 4} \\ \underline{-(x^4 + 0x^3 + 0x^2 - x - 4)} \\ 4x^3 + 0x^2 + 0x - 4 \\ \underline{-(4x^3 + 0x^2 + 0x - 4)} \\ 0 \end{array}$$

$x + 4$

5) $(2x^4 - 3x^3 - 8x^2 + 5x + 4) \div (2x + 1)$

$$\begin{array}{r} x^3 - 2x^2 - 3x + 4 \overline{) 2x^4 - 3x^3 - 8x^2 + 5x + 4} \\ \underline{-(2x^4 + x^3)} \\ -4x^3 - 8x^2 \\ \underline{-(-4x^3 - 2x^2)} \\ -6x^2 + 5x \\ \underline{-(-6x^2 - 3x + 4)} \\ 8x + 4 \\ \underline{-(8x + 4)} \\ 0 \end{array}$$

$x^3 - 2x^2 - 3x + 4$

6) $(x^4 + 3x^2 + x + 4) \div (x + 3)$

$$\begin{array}{r} -3 \overline{) 1 \ 0 \ 3 \ 1 \ 4} \\ \underline{\downarrow -3 \ 9 \ -36 \ 105} \\ 1 \ -3 \ 12 \ -35 \ \boxed{109} \\ x^3 \ x^2 \ x \ c \ R \end{array}$$

$x^3 - 3x^2 + 12x - 35 + \frac{109}{x+3}$

Factor COMPLETELY (Trinomials, Grouping, Sum/Difference of Cubes, Diff of Squares)

7) $8x^3 - 343$ $a = 2x$ $b = 7$

$$(2x-7)(4x^2 + 14x + 49)$$

8) $x^4 + 7x^2 + 6$

$$(x^2+6)(x^2+1)$$

x^4	1	x^2
$6x^2$	6	

6

9) $(25x^3 - 100x^2)(x+4)$

$$25x^2(x-4) - 1(x-4)$$

$$(25x^2-1)(x-4)$$

$$(5x+1)(5x-1)(x-4)$$

10) $(4x^3 + 8x^2)(9x - 18)$

$$4x^2(x+2) - 9(x+2)$$

$$(4x^2-9)(x+2)$$

$$(2x-3)(2x+3)(x+2)$$

11) $-4x^4 - 500x$ $a = x$ $b = 5$
 $-4x(x^3 + 125)$

$$-4x(x+5)(x^2 - 5x + 25)$$

12) $(x^5 - 3x^4)(16x + 48)$

$$x^4(x-3) - 16(x-3)$$

$$(x^4-16)(x-3)$$

$$(x^2+4)(x^2-4)(x-3)$$

$$(x^2+4)(x+2)(x-2)(x-3)$$

13) $x^2(x+3) - 1(x+3)$

$$(x^2-1)(x+3)$$

$$(x+1)(x-1)(x+3)$$

14) $5x^4 + 45x^3 + 100x^2$

$$5x^2(x^2 + 9x + 20)$$

$$5x^2(x+5)(x+4)$$

Factor/Remainder Theorem

15) Is $x + 3$ a factor of $x^3 + 4x^2 - x - 1$?

$$\begin{array}{r} -3 \overline{) 1 \ 4 \ -1 \ -1} \\ \underline{-3 \ -3 \ 12} \\ 1 \ 1 \ -4 \ 11 \end{array} \rightarrow \text{No, it's not a Remainder}$$

16) 15. Is $x - 4$ a factor of $x^3 - 6x^2 + 5x + 12$?

$$\begin{array}{r} 4 \overline{) 1 \ -6 \ 5 \ 12} \\ \underline{4 \ -8 \ -12} \\ 1 \ -2 \ -3 \ 0 \end{array} \rightarrow \text{yes, remainder is 0}$$

17) If $(x - 4)$ is a factor, what are all of the zeros of the polynomial? What are all the factors?

$$f(x) = x^3 - 13x - 12$$

$$\begin{array}{r} 4 \overline{) 1 \ 0 \ -13 \ -12} \\ \underline{4 \ 16 \ 12} \\ 1 \ 4 \ 3 \ 0 \end{array}$$

$x^2 \quad x \quad c$

$$x^2 + 4x + 3$$

$$(x + 3)(x + 1)$$

Zeros: $x = -3 \quad x = -1 \quad x = 4$

factors: $(x + 3)(x + 1)(x - 4)$

Writing Polynomials Given Zeros

18) Write a polynomial equation given the zeros of the function are $\frac{2}{5}$, -3 , and 1 .

$$\begin{array}{l} (x - \frac{2}{5})(x + 3)(x - 1) \\ \boxed{(5x - 2)(x + 3)(x - 1)} \end{array} \rightarrow \begin{array}{r} 5x - 2 \\ x \overline{) 5x^2 - 2x} \\ \underline{15x - 6} \\ 5x^2 + 13x - 6 \\ x \overline{) 5x^3 + 13x^2 - 6x} \\ \underline{-5x^3 - 13x^2 + 6x} \\ -6 \end{array} \rightarrow \boxed{5x^3 + 8x^2 - 19x + 6}$$

19) Write a polynomial equation given the zeros of the function are $3 \pm 2i$ and 5 .

$$(x - 3 - 2i)(x - 3 + 2i)(x - 5)$$

x	x^2	$-3x$	$-2i$
-3	$-3x$	9	$6i$
$2i$	$2xi$	$-6i$	$-4i^2$

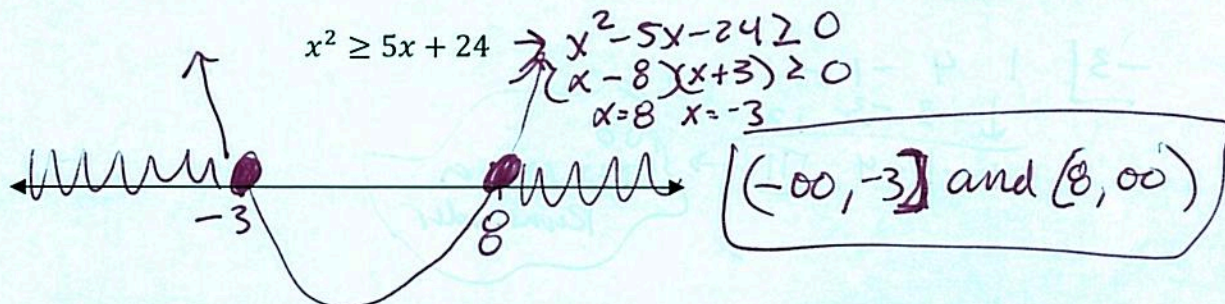
 $x^2 - 6x + 13$

x	x^2	$-6x$	$+13$
x	x^3	$-6x^2$	$+13x$
-5	$-5x^2$	$+30x$	-65

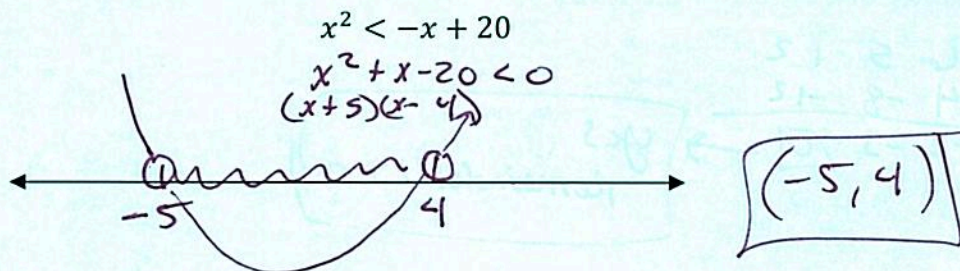
 $\boxed{x^3 - 11x^2 + 43x - 65}$

Polynomial Inequalities

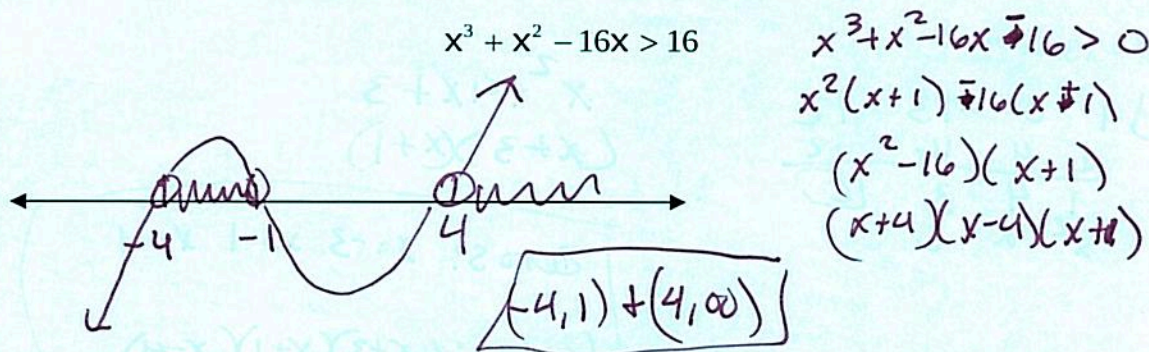
20) Graph the solution set on the given number line for the following inequality:



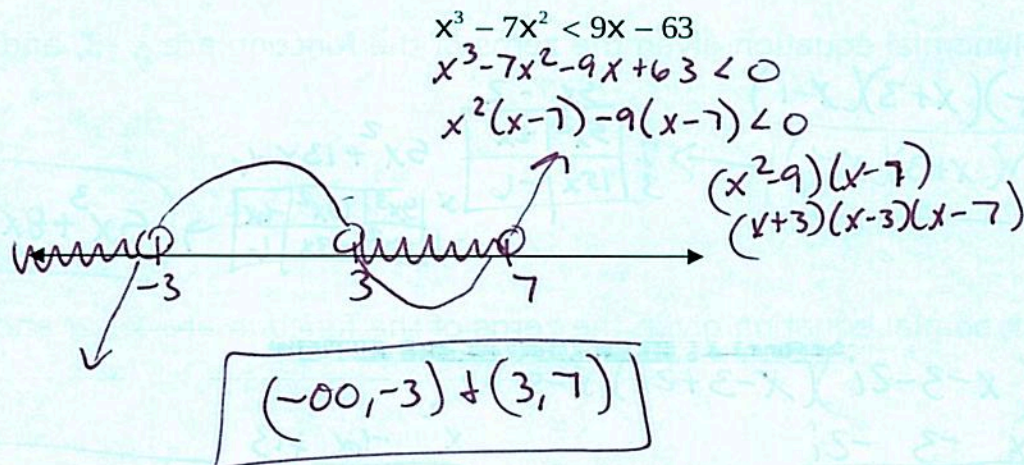
21) Graph the solution set on the given number line for the following inequality:



22) Graph the solution set on the given number line for the following inequality:



23) Graph the solution set on the given number line for the following inequality:



Regression: On the class practice sheet done in class!