PROBLEM 3 Inverse by Composition

You know that when the domain is restricted to $x \geq 0$, the function $f(x) = \sqrt{x}$ is the inverse of the power function $g(x) = x^2$. You also know that the function $h(x) = \sqrt[3]{x}$ is the inverse of the power function $q(x) = x^3$.

The process of evaluating one function inside of another function is called the **composition of functions**. For two functions f and g, the composition of functions uses the output of $g(x)$ as the input of $f(x)$. It is notated as $(f \circ g)(x)$ or $f(g(x))$.

To write a composition of the functions $g(x) = x^2$ and $f(x) = \sqrt{x}$ when the domain of $g(x)$ is restricted to $x \geq 0$, substitute the value of one of the functions for the argument, x, of the other function.

$$f(x) = \sqrt{x} \qquad\qquad g(x) = x^2$$

$$f(g(x)) = \sqrt{x^2} = x, \text{ for } x \geq 0$$

You can write the composition of these two functions as $f(g(x)) = x$ for $x \geq 0$.

1. Determine $g(f(x))$ for the functions $g(x) = x^2$ and $f(x) = \sqrt{x}$ for $x \geq 0$.

$g(f(x)) = (\sqrt{x})^2 = x$ ✓

$f(g(x)) = \sqrt{x^2} = x$ ✓ yes!

If $f(g(x)) = g(f(x)) = x$, then $f(x)$ and $g(x)$ are inverse functions.

2. Are $f(x)$ and $g(x)$ inverse functions? Explain your reasoning.

✓

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3. Algebraically determine whether each pair of functions are inverses. Show your work.

a. Verify that $h(x) = \sqrt[3]{x}$ is the inverse of $q(x) = x^3$.

$h(q(x)) = \sqrt[3]{(x^3)} = x$

$q(h(x)) = (\sqrt[3]{x})^3 = x$ yes!

b. Determine if $k(x) = 2x^2 + 5$ and $j(x) = -2x^2 - 5$ are inverse functions.

$k(j(x))$ $2(2x^2-5)^2+5$ No!

$2[(2x^2-5)(2x^2-5)]+5$

$2[4x^4-10x^2-10x^2+25]+5$

$2(4x^4-20x^2+25)+5$ $8x^4-40x^2+55$

4. Mike said that all linear functions are inverses of themselves because $f(x) = x$ is the inverse of $g(x) = x$.

Is Mike correct? Explain your reasoning.

No $f(x) = x$ $g(x) = x+2$

$f(g(x)) = x+2$ No!

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13.4 Verifying Inverses by Compositions of Functions

Determine $(f \circ g)(x)$ *and* $(g \circ f)(x)$ for each pair of functions $f(x)$ *and* $g(x)$. State whether the functions are inverses of each other or not.

A. $f(x) = 5^{x-1}$ $g(x) = x + 3$

$f(g(x)) = 5^{x+3-1} = 5^{x+2}$

$g(f(x)) = 5^{x-1} + 3$

No!

B. $f(x) = 0.5x + 1.5$ $g(x) = 2x - 3$

$f(g(x)) = .5(2x-3) + 1.5 = x - 1.5 + 1.5 = x$ ✓

$g(f(x)) = 2(\frac{1}{2}x + 1.5) - 3 = x + 3 - 3 = x$ ✓

yes!

C. $f(x) = 2x^2 - x$ $g(x) = \sqrt{x}$

$f(g(x)) = 2(\sqrt{x})^2 - x = 2x - x = x$!

$g(f(x)) = \sqrt{2x^2 - x}$

No!

D. $f(x) = (x + 6)^2$ $g(x) = \sqrt{x} - 6$

$f(g(x)) = (\sqrt{x} - 6 + 6)^2 = (\sqrt{x})^2 = x$

$g(f(x)) = \sqrt{(x+6)^2} - 6 = x + 6 - 6 = x$ ✓

yes!

E. $f(x) = 2\sqrt{x - 5}$ $g(x) = \frac{1}{4}x^2 - 5$

$f(g(x)) = 2\sqrt{(\frac{1}{4}x^2 - 5) - 5} = 2\sqrt{\frac{1}{4}x^2 - 10}$

No!

$g(f(x)) = \frac{1}{4}(2\sqrt{x-5})^2 - 5$

F. $f(x) = 4x + 6$ $g(x) = \frac{x-6}{4}$

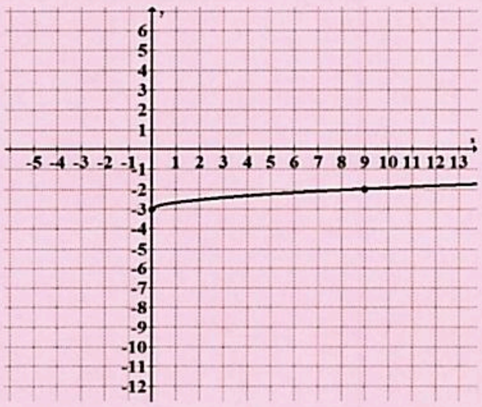
$f(g(x)) = \cancel{4}\left(\frac{x-6}{\cancel{4}}\right) + 6 = x - 6 + 6 = x$

$g(f(x)) = \frac{(4x+6) - 6}{4} = \frac{4x}{4} = x$

yes!!

Radical functions and their Inverses from Graphs

A. Determine the equation of the given graph. You must calculate the "a" value.
B. Determine the inverse function of the given graph.

1. 

A. $y = \frac{1}{3}\sqrt{x} - 3$

$y = a\sqrt{x} - 3$

$-2 = a\sqrt{9} - 3$

$+3 \qquad +3$

$\frac{1}{3} = \frac{a3}{3} \quad a = 1/3$

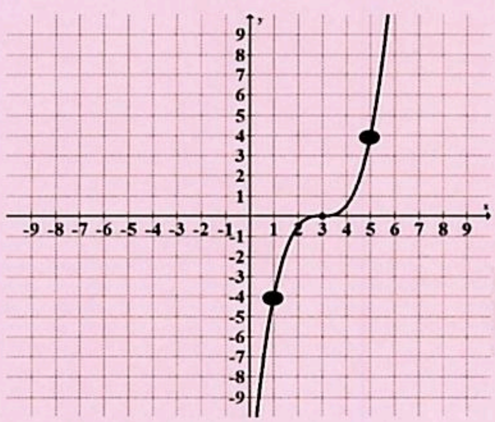
B. $y = (3(x+3))^2$ or $y = (3x+9)^2$

$x = \frac{1}{3}\sqrt{y} - 3$

$x + 3 = \frac{1}{3}\sqrt{y}$

$3(x+3) = \sqrt{y}$

$y = (3(x+3))^2$

2. 

A. $y = \frac{1}{2}(x-3)^3$

$y = a(x-3)^3$

$4 = a(5-3)^3$

$4 = a(2)^3$

$\frac{4}{8} = \frac{8a}{8}$

$a = 1/2$

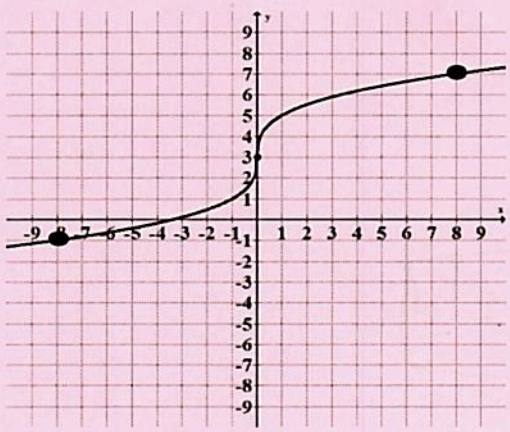
B. $y = \sqrt[3]{2x} + 3$

$x = \frac{1}{2}(y-3)^3$

$2x = (y-3)^3$

$\sqrt[3]{2x} = y - 3$

$y = \sqrt[3]{2x} + 3$

3. 

A. $y = 2\sqrt[3]{x} + 3$

$y = a\sqrt[3]{x} + 3$

$7 = a\sqrt[3]{8} + 3$

$-3 \qquad -3$

$4 = a\sqrt[3]{8}$

$4 = a(2) \quad a = 2$

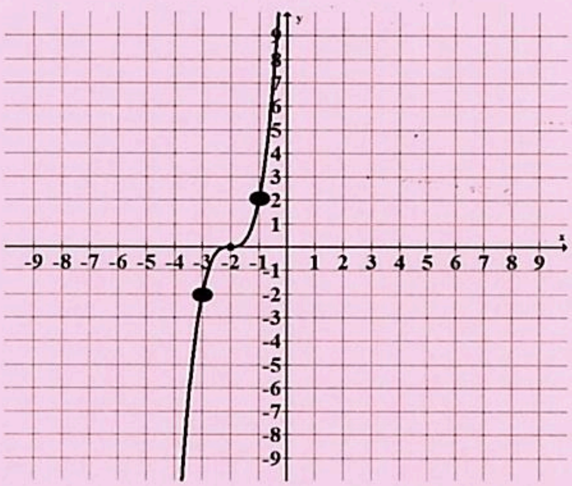
B. $y = \left(\frac{x-3}{2}\right)^3$

$x = 2\sqrt[3]{y} + 3$

$\frac{x-3}{2} = \frac{2\sqrt[3]{y}}{2}$

$\frac{x-3}{2} = \sqrt[3]{y} \quad y = \left(\frac{x-3}{2}\right)^3$

4.



A. $y = 2(x+2)^3$

$y = ax^3 \rightarrow y = a(x+2)^3$

$2 = a(-1+2)^3$

$2 = a(+1)^3$

$a = 2$

B. $y = \sqrt[3]{\frac{1}{2}x} - 2$

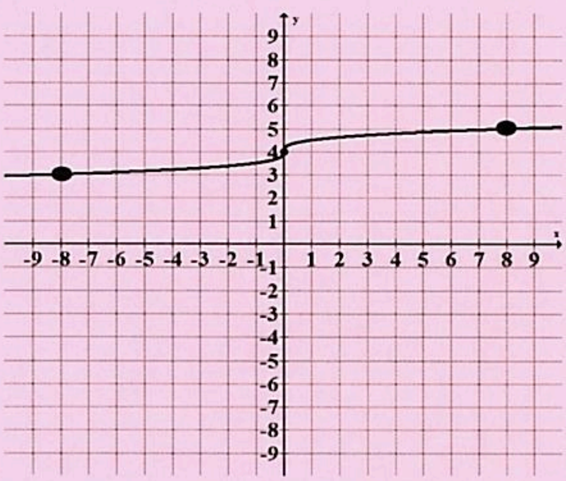
$x = 2(y+2)^3$

$\frac{1}{2}x = (y+2)^3$

$\sqrt[3]{1/2x} = y+2$

$y = \sqrt[3]{1/2x} - 2$

5.



A. $y = 1/2\sqrt[3]{x} + 4$

$y = a\sqrt[3]{x} + 4$

$5 = a\sqrt[3]{8} + 4$

$-4 \qquad -4$

$1 = a(2) \qquad a = 1/2$

B. $y = (2(x-4))^3$ or $y = (2x-8)^3$

$x = 1/2\sqrt[3]{y} + 4$

$x - 4 = 1/2\sqrt[3]{y}$

$2(x-4) = \sqrt[3]{y}$

$y = (2(x-4))^3$