PAP Algebra II

Name: ________________

Transformations of Square Root and Cube Root Summary

$y = a\sqrt{b(x-c)} + d$ $\qquad$ $y = a\sqrt[3]{b(x-c)} + d$

a >1: V. Stretch $\qquad$ b>1: h. comp $\qquad$ -c: Right

0<a<1: V. comp. $\qquad$ 0<b<1: h. Stretch $\qquad$ +c : left

a<0: V. flip $\qquad$ b< 0: h. flip $\qquad$ -d: Down

+d: Up

PAP Algebra II

Name: ________________

Transformations of Square Root and Cube Root Summary

$y = a\sqrt{b(x-c)} + d$ $\qquad$ $y = a\sqrt[3]{b(x-c)} + d$

a >1: ____________ $\qquad$ b>1: ____________ $\qquad$ -c: ____________

0<a<1: ____________ $\qquad$ 0<b<1: ____________ $\qquad$ +c : ____________

a<0: ____________ $\qquad$ b< 0: ____________ $\qquad$ -d: ____________

+d: ____________

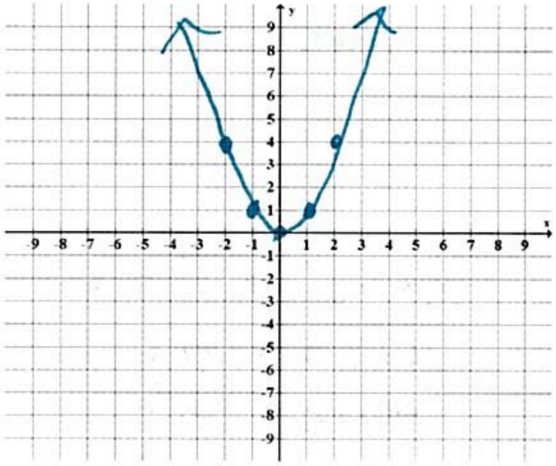
key

10.3 Explore /explain Square and Cube Root Transformations

Quadratic & Square Root Functions

$f(x) = x^2$

X	Y
-2	4
-1	1
0	0
1	1
2	4
3	9



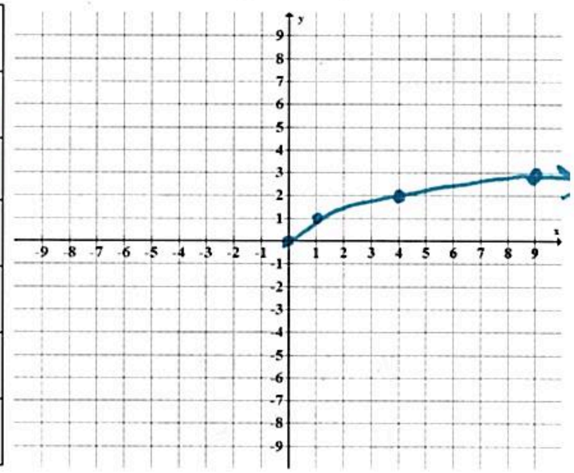
Domain: $\mathbb{R}$ Range: $y \geq 0$

x-int: (0,0) y-int: (0,0)

min: (0,0) max: None

$f(x) = \sqrt{x}$

X	Y
-4	error
-1	error
0	0
1	1
4	2
9	3



Domain: $x \geq 0$ Range: $y \geq 0$

x-int: (0,0) y-int: (0,0)

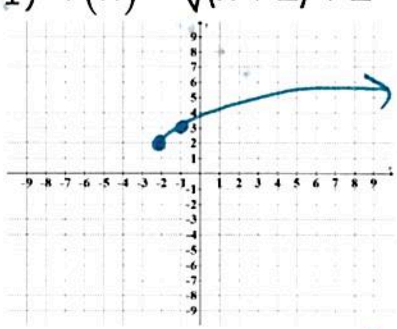
min: (0,0) max: None

Why do you think the table gives you an error for a y value for $f(x) = \sqrt{x}$?

Negatives

For the following list the transformations compared to the parent function, find the domain, range, maximum, and minimum. Sketch if needed.

1) $f(x) = \sqrt{(x+2)} + 2$

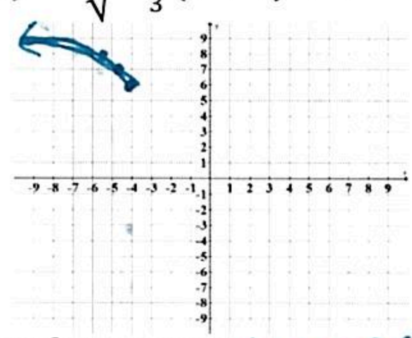


List transformations: left 2; up 2

Domain: $x \geq -2$ Maximum: None

Range: $y \geq 2$ Minimum: (-2,2)

2) $f(x) = \sqrt{-\frac{1}{3}(x+4)} + 6$



0	0
1	1
4	2

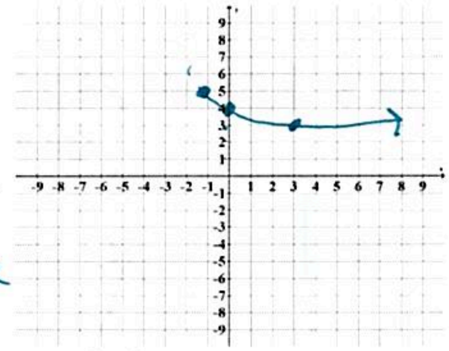
-4
$-\frac{13}{3}$
$-\frac{1}{3}$
$-\frac{4}{3}$

List transformations: horiz Reflection left 4 up 6

Domain: $x \leq -4$ Maximum: None

Range: $y \geq 6$ Minimum: -4,6

3) $f(x) = -\sqrt{(x+1)} + 5$



List transformations: Vert. Reflection ~~right~~ left 1 up 5

Domain: $x \geq -1$

Range: $y \leq 5$

Maximum: $-1, 5$

Minimum: None

For the following functions find $f^{-1}(x)$. Note domain restrictions.

4) $f(x) = \sqrt{x-2} + 4$

$x = \sqrt{y-2} + 4$

$x - 4 = \sqrt{y-2}$

$(x-4)^2 = y - 2$

$\boxed{y = (x-4)^2 + 2 \quad x \geq 4}$

5) $f(x) = -\sqrt{x+7} - 1$

$x = -\sqrt{x+7} - 1$

$x + 1 = -\sqrt{\cancel{x}y+7}$

$-(x+1) = \sqrt{\cancel{x}y+7}$

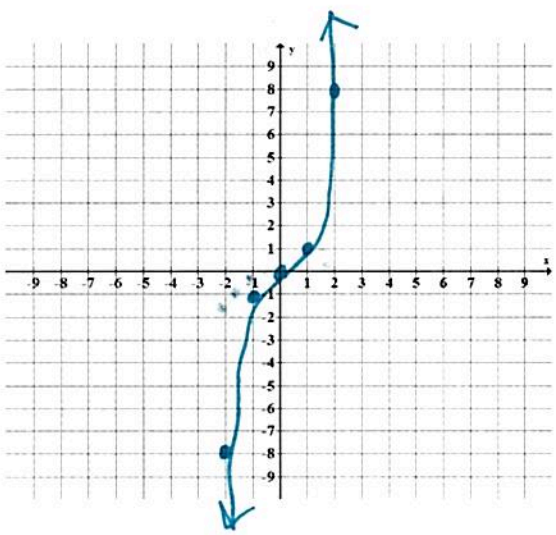
$(-(x+1))^2 = \cancel{x}y+7$

$\boxed{y = (-(x+1))^2 - 7 \quad x \leq -1}$

Cube & Cube Root Functions

$f(x) = x^3$

X	Y
-2	-8
-1	-1
0	0
1	1
2	8



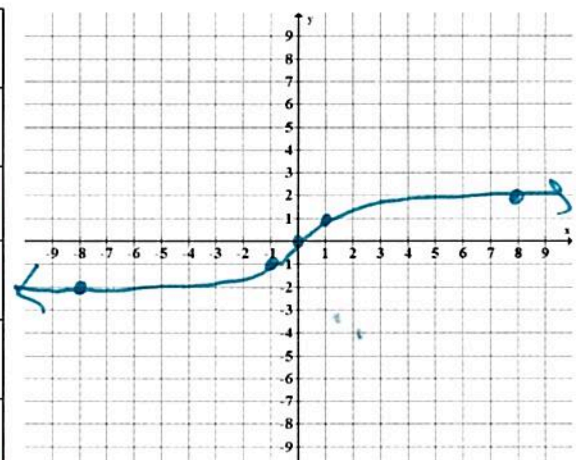
Domain: ℝ Range: ℝ

x-int: (0,0) y-int: (0,0)

min: None max: None

$f(x) = \sqrt[3]{x}$

X	Y
-8	-2
-1	-1
0	0
1	1
8	2



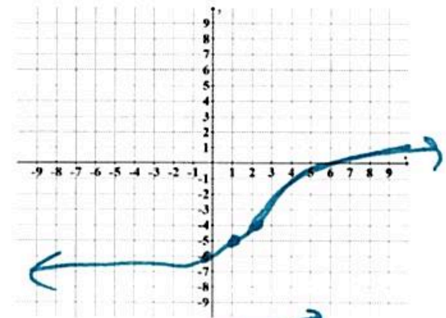
Domain: ℝ Range: ℝ

x-int: (0,0) y-int: (0,0)

min: None max: None

For the following list the transformations compared to the parent function, find the domain, range, maximum, and minimum. Sketch if needed.

1) $f(x) = \sqrt[3]{(x-1)} - 5$



List transformations: Right 1 Down 5

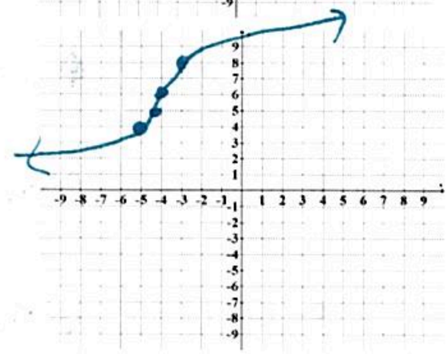
Domain: $\mathbb{R}$

Maximum: None

Range: $\mathbb{R}$

Minimum: None

2) $f(x) = 2\sqrt[3]{(x+4)} + 6$



x	x2+6
-1	-1
0	0
1	1

(x+4)	x2	+6
-5	-2	4
-4	0	6
-3	2	8

List transformations: Vert. Stretch left 4; up 6

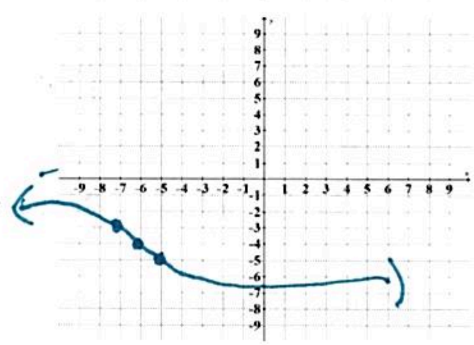
Domain: $\mathbb{R}$

Maximum: None

Range: $\mathbb{R}$

Minimum: None

3) $f(x) = \sqrt[3]{-(x+6)} - 4$



-1	-1
0	0
1	1

-5	1	-5
-6	0	-4
-7	-1	-3

List transformations: horiz Reflect; left 6; Down 4

Domain: $\mathbb{R}$

Maximum: None

Range: $\mathbb{R}$

Minimum None

For the following functions find $f^{-1}(x)$. Note domain restrictions.

4) $f(x) = -2x^3 + 3$

$x = -2y^3 + 3$

$\frac{x-3}{-2} = \frac{-2y^3}{-2}$

$\frac{x-3}{-2} = y^3$

$\boxed{y = \sqrt[3]{\frac{x-3}{-2}}}$

5) $f(x) = \sqrt[3]{x+1} - 4$

$x = \sqrt[3]{y+1} - 4$

$x + 4 = \sqrt[3]{y+1}$

$(x+4)^3 = y + 1$

$\boxed{y = (x+4)^3 - 1}$

Compositions of Functions Notes

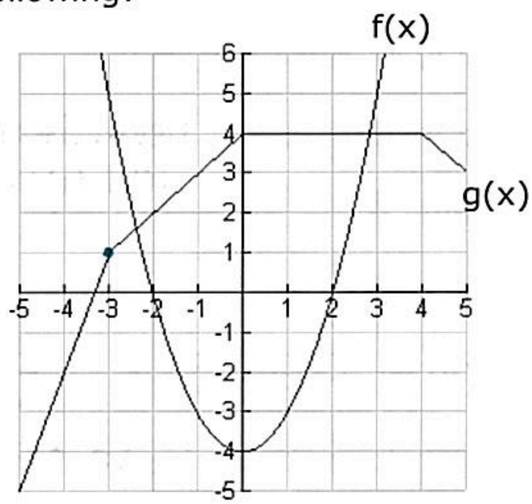
FUNCTION PROPERTIES:

Given functions $f(x)$ and $g(x)$ the following properties hold true:

COMPOSITION: $$(f \circ g)(x) = f(g(x))$$

To compose two functions means to make the output of one function the input of another function.

1. Given the following graph of f(x) and g(x) find the following:



a. $(f \circ g)(-3) =$ $f(1) = \boxed{-3}$

b. $(f \circ g)(-1) =$ $f(3) = \boxed{5}$

c. $(g \circ f)(-3) =$ $g(5) = \boxed{3}$

d. $(f \circ f)(-2) =$ $f(0) = \boxed{-4}$

e. $(g \circ g)(-3) =$ $g(1) = \boxed{4}$

2. Given the functions $f(x) = 4x + 2$, $g(x) = x^2$, $h(x) = 2x - 3$, and $j(x) = \sqrt{x+3}$ find:

a) $j(13) = \sqrt{13+3} = \boxed{4}$

b) $j(2b) = \boxed{\sqrt{2b+3}}$

c) $j(y^2) = \boxed{\sqrt{y^2+3}}$

d) f(3) = $4(3)+2 = \boxed{14}$

e) g(-3) = $(-3)^2 = \boxed{9}$

f) g(5) = $5^2 = \boxed{25}$

g) g(h(1))= $(2(1)-3)^2$
$(-1)^2 = \boxed{1}$

h) h(f(-1))= $4(-1)+2 = \boxed{-2}$
$2(-2)-3$
$-4-3 = \boxed{-7}$

i) j(h(2)) = $2(2)-3 = 1$
$\sqrt{1+3} = \boxed{2}$

j) $j(g(x)) = \sqrt{x^2+3}$

k) $j(f(g(x))) =$ x^2
$4(x^2)+2$
$\sqrt{4x^2+2+3}$
$\boxed{\sqrt{4x^2+5}}$

l) $g(j(h(x))) =$
$g(j(2x-3))$
$g(\sqrt{2x-3+3})$
$g(2x)$
$(2x)^2 = \boxed{4x^2}$

☆

PAP Algebra II Name: Key

Homework: Square & Cube Root Transformations & Intro to Compositions

For the given function equations find the following information. Make a sketch if necessary.

1. $f(x)=2\sqrt{x+5}+3$

Transformations: Vert. Stretch left 5; up 3

Domain: $x \geq -5$

Range: $y \geq 3$

Max value: None

Min value: $(-5,3)$

2. $f(x)=-\sqrt{2(x-2)}+1$

Transformations: V. Reflect. horiz comp. Right 2 up 1

Domain: $x \geq 2$

Range: $y \leq 1$

Max value: $(2,1)$

Min value: None

3. $f(x)=\sqrt[3]{x+3}-4$

Transformations: left 3 down 4

Domain: $x \geq -3$

Range: $y \geq -4$

Max value: None

Min value: None

4. Write an equation using the square root parent function with a minimum value of -3

$y=\sqrt{x}-3$

5. Write an equation using the square root parent function with a maximum value of 4

$y=-\sqrt{x}+4$

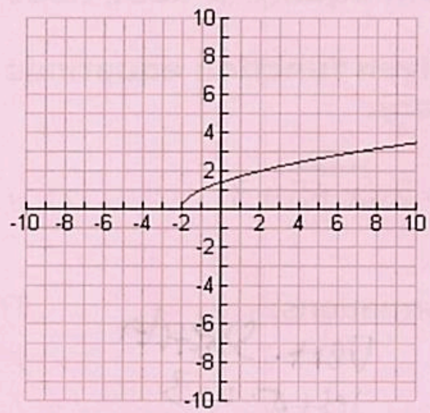
6. Write an equation using the parent function $f(x)=\sqrt{x}$ that has a domain of $-2 \leq x < \infty$ and a range of $4 \leq y < \infty$.

$y=\sqrt{x+2}+4$

7. Write an equation where the parent function $f(x)=\sqrt[3]{x}$ has been shifted to the left 3, shifted down 3 and vertically stretched.

$y=2\sqrt[3]{x+3}-3$

Write an equation from the graph. State the domain & range of the function. Assume a = 1.



8.

Equation: $y = \sqrt{x-5} + 4$

Domain: $x \geq 5$

Range: $y \geq 4$

9.

Equation: $y = \sqrt{x+2}$

Domain: $x \geq -2$

Range: $y \geq 0$

For the following functions find $f^{-1}(x)$. Note domain restrictions.

10. $f(x) = \sqrt[3]{x-4} + 8$

$x = \sqrt[3]{y-4} + 8$

$x - 8 = \sqrt[3]{y-4}$

$(x-8)^3 = y-4$

$\boxed{y = (x-8)^3 + 4}$

12. $f(x) = \sqrt{x-4} - 3$

$x = \sqrt{y-4} - 3$

$x + 3 = \sqrt{y-4}$

$(x+3)^2 = y-4$

$\boxed{y = (x+3)^2 + 4 \quad x \geq -3}$

11. $f(x) = -\sqrt{x-6} - 8$

$x = -\sqrt{y-6} - 8$

$x + 8 = -\sqrt{y-6}$

$(-(x+8)) = \sqrt{y-6}$

$((-(x+8))^2 = y-6$

$\boxed{y = (-(x+8))^2 + 6 \quad x \leq -8}$

13. $f(x) = 3x^3 + 7$

$x = 3y^3 + 7$

$\frac{x-7}{3} = \frac{3y^3}{3}$

$y^3 = \frac{x-7}{3}$

$\boxed{y = \sqrt[3]{\frac{x-7}{3}}}$

14. Given the functions $f(x)=x^2-3$, $g(x)=3x-1$, $h(x)=\frac{1}{x}$ and $j(x)=\sqrt{x-5}$ find:

a) $f(-4)=$ $(-4)^2-3=$
$16-3=\boxed{13}$

b) $f(3x-2)=$ $(3x-2)^2-3$
$(3x-2)(3x-2)-3$
$9x^2-6x-6x+4-3$
$\boxed{9x^2-12x+1}$

c) $g(-8)=$ $3(-8)-1$
$-24-1$
$\boxed{-25}$

d) $g(4x+2)=$
$3(4x+2)-1$
$12x+6-1$
$\boxed{12x+5}$

e) $j(g(x))=$
$\sqrt{(3x-1)-5}$
$\boxed{\sqrt{3x-6}}$

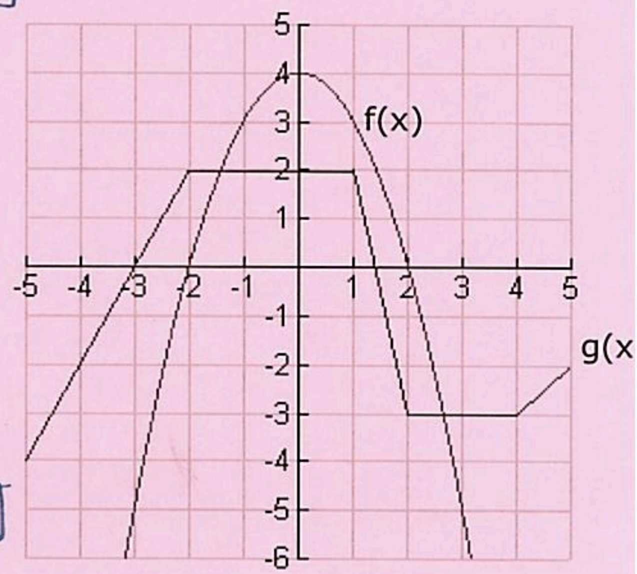
f) $f(g(x))=$
$f(3x-1)$
$(3x-1)^2-3$
$(3x-1)(3x-1)-3$
$9x^2-6x+1-3$
$\boxed{9x^2-6x-2}$

g) $j(g(5))=$
$j(3(5)-1)$
$j(14)=\sqrt{14-5}$
$\sqrt{9}=\boxed{3}$

h) $h(f(g(x)))=$
$h(f((3x-1)))$
$h((3x+1)^2-3)$
$h((3x-1)(3x-1)-3)$
$h(9x^2-6x+1-3)$
$h(9x^2-6x-2)$
$\boxed{\dfrac{1}{9x^2-6x-2}}$

15. Given the following graphs of f(x) and g(x) find the following:

a) $(f\circ g)(1)=f(2)=\boxed{0}$ **b)** $(g\circ f)(0)=g(4)=\boxed{-3}$



c) $(f\circ g)(-3)=f(0)=\boxed{4}$ **d)** $(g\circ f)(3)=g(-3)=\boxed{0}$

e) $(g\circ g)(1)=g(2)=\boxed{-3}$ **f)** $(f\circ f)(-1)=f(3)=\boxed{-5}$