Explore: Inverses Graphically

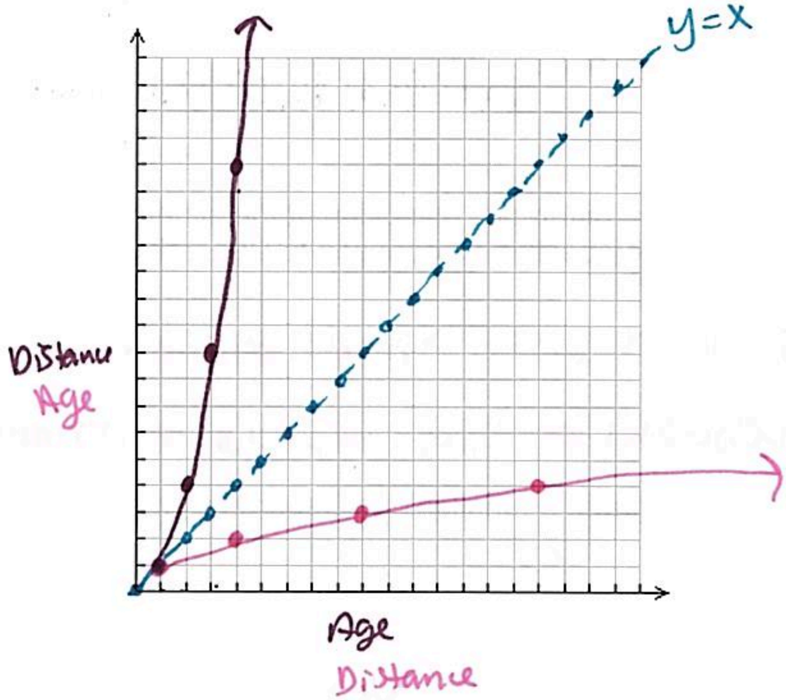
The inverse is a Reflection of a graph over the line $y = x$.

So, to take the inverse, we Switch the x- and y-values or the domain and range.

1) Sharon recorded some data from children in the DFW area on how their age in years compared to how far they hit a golf ball in m. She was thinking that the distance depended on the age. Graph the data. (hint: count by 2.5)

Age	Distance
1	1
2	4
3	9
4	16
5	25
6	36
7	49

Dist	Age
1	1
4	2
9	3
16	4
25	5



a. What is the domain of Sharon's graph? [1, 7]

b. What is the range? [1, 49]

2) Kenny thought that the age depended on the distance. He took Sharon's data and changed it so that the x-value represents the distance and the y-value represents the age. Graph this data on the same grid, but in a different color.

a. What is the domain of Kenny's graph? [1, 49]

b. What is the range? [1, 7]

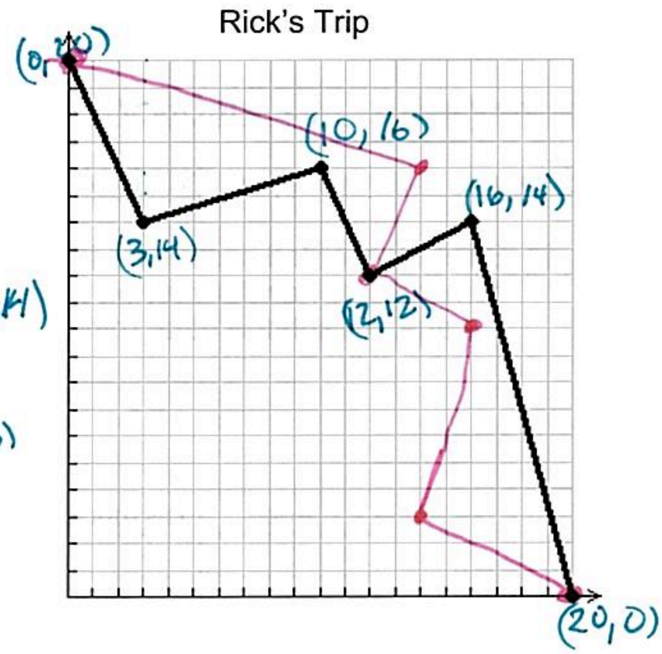
3) What kind of relationship exists between Sharon's and Kenny's way of looking at the data?

Inverses

4) Over what line is the graph of the data reflected? Graph this line in a third color.

$y = x$

5) The following graph represents Rick's trip home from a friend's house with distance from home in terms of time. Deborah thinks that it should be time in terms of distance. Plot Deborah's graph on the same plane. What is the relationship between these two graphs?



Rick's Key Points: (0, 20); (3,14); (10,16); (12,12); (16,14) (20,0)

Deborah's Key Points: (20,0) (14,3) (16,10) (12,12) (14,16) (0,20)

6) What is the difference in looking at the situation from Rick's perspective as opposed to Deborah's perspective?

Rick → Dist. Depends on Time

Deborahs → Time Depends on Distance

7) Is Rick's graph a function? Is Deborah's graph a function?

Yes NO

8) Does Rick's graph pass the vertical line test? Does his graph pass the horizontal line test?

Yes NO

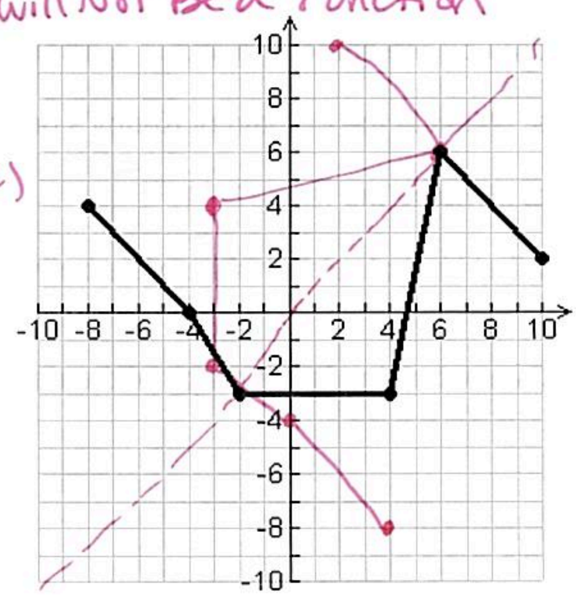
9) If a graph passes the horizontal line test, will its inverse be a function?

NO

10) What does it mean if the graph does not pass the horizontal line test?

Its inverse will Not Be a Function

11) Graph the inverse of this function; graph the line of reflection as well.



Key Points: (-8,4) (-4,0) (-2,-3) (4,-3) (6,6) (10,2)

Inverse Key Points: (4,-8) (0,-4) (-3,-2) (-3,4) (6,6) (2,10)

12) Is the inverse a function? Justify your answer.

NO ~~Original~~ fails HLT

key

Elaborate Inverses - Algebraically

Jacques is a French foreign exchange student. Since Celsius is the temperature scale in France, Jacques uses the function $C(F)=\frac{5}{9}(F-32)$ (or $C=\frac{5}{9}(F-32)$) for converting from degrees Fahrenheit, F, to degrees Celsius, C. He derived the function from the freezing point (32, 0) and the boiling point (212, 100), where each ordered pair is (F, C) or (Fahrenheit, Celsius).

1. What are the independent and dependent variables in $C=\frac{5}{9}(F-32)$? Explain.

   Independent: Fahrenheit Dependent: Celsius

2. Evaluate C(90) = 194° Write the ordered pair (F, C) and explain its meaning in the context of this problem.

   $90 = \frac{5}{9}(F-32)$ (194, 90)

   $162 = 5 - 32$

Now, you and Jacques are taking a trip to Paris, which uses the Celsius temperature scale. You need to write a function that will give an output of degrees Fahrenheit when using an input of degrees Celsius.

. Now, what are the independent and dependent variables for this new function?

   Independent: Celsius Dependent: fahrenheit

4. How are this function and the original function related? Inverses

5. Write the function for the new situation.

$$C = \frac{5}{9}(F-32)$$

$$\frac{9}{5}C = F - 32$$

$$\boxed{F = \frac{9}{5}C + 32}$$

6. Substitute some values to check your function and make sure it is working.

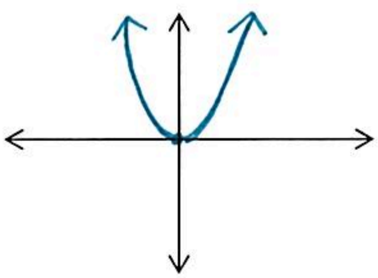
Elaborate Inverses - Algebraically

When we write an equation in terms of x and y, x is the independent variable and y is the dependent variable. Thus, the steps for finding an inverse of an equation written in terms of x and y are:

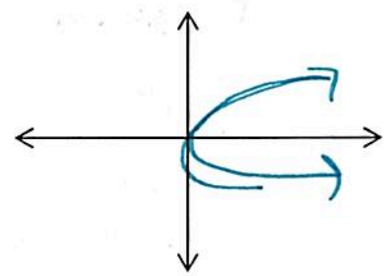
- If application problem, solve the equation for independent variable.
- If Non-application problem – switch "x" and "y", solve for y.
- You must think about Domain Restrictions to ensure that the inverse will be a function.

Domain Restrictions:

Sketch $y=x^2$



Reflect graph over y=x to sketch the inverse. Is it a function?



Not Function

Find the inverse function, $f^{-1}(x)$, of the following functions.

1. $f(x) = 2x^2 + 4$ with domain $x \geq 0$

$$x = 2y^2 + 4$$
$$\frac{x-4}{2} = \frac{2y^2}{2}$$
$$\sqrt{y^2} = \sqrt{\frac{x-4}{2}}$$
$$\boxed{y = \sqrt{\frac{x-4}{2}}}$$

2. $f(x) = 3(x - 4)^2 - 2$ with domain $x \geq 4$

$$x = 3(y-4)^2 - 2$$
$$\frac{x+2}{3} = \frac{\cancel{3}(y-4)^2}{\cancel{3}}$$
$$\frac{x+2}{3} = (y-4)^2$$
$$\sqrt{\frac{x+2}{3}} = y-4$$
$$\boxed{y = \sqrt{\frac{x+2}{3}} + 4}$$

3. $f(x)= \frac{1}{3}x^2 - 3$ with domain $x \geq 0$

$$x = \frac{1}{3}x^2 - 3$$
$$x+3 = \frac{1}{3}y^2$$
$$3(x+3) = y^2$$
$$\boxed{\sqrt{3(x+3)} = y}$$

4. $f(x)= \frac{2}{5}(x+3)^2$ with domain $x \leq -3$

$$x = \frac{2}{5}(y+3)^2$$
$$\frac{5}{2}x = (y+3)^2$$
$$\sqrt{\frac{5}{2}x} = y+3$$
$$\boxed{y = -\sqrt{\frac{5}{2}x} - 3}$$

Evaluate: Inverses and Radical Functions Name: Key

Sandra owns a painting that has a value represented by the function V(T) = 600 + 100T, where T is the time in years.

1. What are the independent and dependent variables?

Independent: Time Dependent: Value

Domain: $x > 0$ Range: $y \geq 600$,

2. What is the value of the painting after 4 years? 8 years?

$V(t) = 600 + 100(4)$
$V(t) = 1000$

$V(t) = 600 + 100(8)$
1400

Sandra is curious to know when the painting will be worth $1500, $2000, or $7000.

3. Since Sandra wants to solve the problem several times, she has decided to rewrite the function where T is the dependent variable and the value, V, is the independent variable. What is the relationship between this new function and the function V = 600 + 100T?

Inverses

4. Write the new function. T(V) = $\frac{V-600}{100}$

5. Fill in the table that supplies a number of different values for the painting.

Value (V)	Time in years (T)
1000	4
2500	19
5000	44
10000	54

Find the inverse function, $f^{-1}(x)$, of each of function.

6. $f(x) = 3x^2 - 6$ with domain $x \leq 0$

$x = 3y^2 - 6$

$\frac{x+6}{3} = \frac{3y^2}{3}$

$\sqrt{y^2} = \sqrt{\frac{x+6}{3}}$

$\boxed{y = -\sqrt{\frac{x+6}{3}}}$ $x \geq -6$

7. $f(x) = \frac{-5}{3}x^2 + 8$ with domain $x \geq 0$

$x = -\frac{5}{3}y^2 + 8$

$x - 8 = -5/3\, y^2$

$-\frac{3}{5}(x-8) = y^2$

$\boxed{y = \sqrt{-\frac{3}{5}(x-8)}}$

Evaluate: Inverses and Radical Functions Name: ____________________

8. f(x)= $2(x-4)^2-5$ with domain x≥4

$x=2(y-4)^2-5$

$\frac{x+5}{2}=\frac{2(y-4)^2}{2}$

$\frac{x+5}{2}=(y-4)^2$

$\sqrt{\frac{x+5}{2}}=y-4$

$\boxed{y=\sqrt{\frac{x+5}{2}}+4}$

Identify the needed restricted domain for the quadratic function, then determine the inverse function.

9. $y=(x-1)^2$

$x=(y-1)^2$

$\sqrt{x}=y-1$

$\boxed{y=\sqrt{x}+1 \quad x\geq 0}$

10. $y=(x+14)^2+7$

$x=(y+14)^2+7$

$x-7=(y+14)^2$

$\sqrt{x-7}=y+14$

$\boxed{y=\sqrt{x-7}-14 \quad x\geq -7}$

11. $y=9x^2-5$

$x=9y^2-5$

$\frac{x+5}{9}=\frac{9y^2}{9}$

$\frac{x+5}{9}=y^2$

$\boxed{y=\sqrt{\frac{x+5}{9}} \quad x\geq -5}$

12. $y=x^2+1$

$x=y^2+1$

$x-1=y^2$

$\boxed{y=\sqrt{x-1} \quad x\geq 1}$